1 Preview

1.1 Some Definitions

Definition 1.1.1 (GDP)**.** GDP measures most of the undiminished production in a given time frame such that

$$GDP_{nominal} = \sum_i P_{i,t} Q_{i,t}$$
$$GDP_{real} = \sum_i P_{i,0} Q_{i,t}$$

where $t = 0$ is the based year.

Definition 1.1.2 (CPI: consumer price index)**.**

$$CPI_t = \frac{\sum_i P_{i,t} Q_{i,0}}{\sum_i P_{i,0} Q_{i,0}}$$

Definition 1.1.3 (PGDP: GDP deflator)**.**

$$PGDP = \frac{GDP_{nominal}}{GDP_{real}} = \frac{\sum_i P_{i,t} Q_{i,t}}{\sum_i P_{i,0} Q_{i_t}}$$

Definition 1.1.4 (Inflation Rate)**.** We can conclude the inflation rate by two different approaches using CPI or PGDP.

$$\text{Inflation rate} := \pi_{t,CPI} = \frac{CPI_t - CPI_{t-1}}{CPI_P t - 1}$$
$$\text{Inflation rate} := \pi_{t,PGDP} = \frac{PGDP_t - PGDP_{t-1}}{PGDP_{t-1}}$$

where t is the year for which the inflation rate is asked.

Definition 1.1.5 (National-income identity)**.** At equilibrium

$$Y^s = Y^d = C + I + G + NX$$

Where

$$Y^s = Af(L, K)$$

in LR

Definition 1.1.6 (Savings)**.**

i. Public Saving is defined: $S_{public} = T - G - (tr)$

ii. Private Saving is defined: $S_{private} = Y - C - T$

Remark. In particular,

$$C = C_0 + C_1(Y - T).$$

Theorem 1.1.6.1 (Real wage/rent)**.**

i. Real Wage:

$$\frac{W}{P} = MP_L$$

assuming profit max.

ii. Real Rental Price:

$$\frac{R}{P} = MP_K$$

assuming profit max.

Proof. Assume profit maximization. Let Π be profit of the economy. Then,

$$\begin{aligned}
\Pi &= P \cdot Y - W \cdot L - R \cdot K \\
&\implies \max(\Pi) \iff \frac{\partial \Pi}{\partial K} = 0 \text{ and } \frac{\partial \Pi}{\partial L} = 0 \\
&\implies \frac{\partial}{\partial K}(P \cdot Y - W \cdot L - R \cdot K) = 0 \text{ and } \frac{\partial}{\partial L}(P \cdot Y - W \cdot L - R \cdot K) = 0 \\
&\implies \frac{R}{P} = MP_K \text{ and } \frac{W}{P} = MP_L && \text{as required.}
\end{aligned}$$

□

Definition 1.1.7 (Growth Accounting Function)**.**

$$\frac{\Delta Y}{Y} = \alpha \frac{\Delta K}{K} + (1 - \alpha)\frac{\Delta L}{L} + \frac{\Delta A}{A}.$$

where α is capital's income share s.t.

$$\alpha = \frac{MP_K \cdot K}{Y} \quad \text{and } 1 - \alpha = \frac{MP_L \cdot L}{Y}.$$

In particular,

$$Y = MP_K \cdot K + MP_L \cdot L.$$

Theorem 1.1.7.1 (growth rate of a ratio)**.** If $B = \frac{C}{D}$, then

$$\frac{\Delta B}{B} = \frac{\Delta C}{C} - \frac{\Delta D}{D}.$$

Proposition 1.1.7.2 (real standard of living)**.** $AP_L = \frac{Y}{L}$ measures the average real standard of living, of which the growth relies on both K and A.

Theorem 1.1.7.3 (Euler's Theorem)**.** Given a CRTS production function $Y = A \cdot f(K, L)$,

$$\begin{aligned} Y &= \frac{\partial Y}{\partial K}K + \frac{\partial Y}{\partial L}L \\ &\iff Y = MP_K K + MP_L L \\ &\iff Y = \frac{R}{P} \cdot K + \frac{W}{P} \cdot L. \\ &\iff 1 = \left[\frac{\frac{R}{P} \cdot K}{Y}\right] + \left[\frac{\frac{W}{P} \cdot L}{Y}\right]. \end{aligned}$$

This implies that in the LR, there is no economic profit.

Definition 1.1.8 (Fisher Equation)**.** Let i be nomial interest rate, r be real interest rate and π be inflation rate, then

$$\begin{aligned} i &= r + \pi \\ &\iff r = i - \pi \end{aligned}$$

2 The Classical Model of the Closed Economy in Long Run

2.1 Solow Long-Run Model

The Solow Model has 7 Axioms

$A_{\mathcal{S}}1.$ $Y = A \cdot F(K, L)$

$A_{\mathcal{S}}2.$ fixed tech and L

$A_{\mathcal{S}}3.$ Positive diminishing $MP_K \wedge MP_L$. K and L are complementary. Constand Return to Scale. Suppliers are perfectly competitive. Thus,

$$\lambda \cdot Y = A \cdot F(\lambda k, \lambda L), \forall \lambda > 0.$$

Choose

$$\lambda = \frac{1}{L} > 0,$$

Then,

$$\frac{Y}{L} = A \cdot f(\frac{K}{L}) \equiv \tilde{Y} = A \cdot f(\tilde{K})$$

i.e., in Cobb-Donglas terms,

$$\tilde{Y} = A \cdot \tilde{K}^{\alpha}.$$

$A_{\mathcal{S}}4.$ Closed Economy

$A_{\mathcal{S}}5.$ No Government

$$Y = C + I$$

$A_{\mathcal{S}}6.$ For $s \in (0,1)$ s.t. $s :=$ marginal propensity to save (MPS),

$$S = s \cdot Y.$$

It then follows that

$$S_{National} = Y - C = s \cdot Y \implies C = (1 - s)Y.$$

In particular,

$$\begin{aligned} Y &= C + S && \text{as MPS + MPC = 1} \\ \iff Y &= C + I && \text{National Income id} \\ \iff S &= I. \end{aligned}$$

Thus by construction

$$S = I$$

i.e., this model is at least in LR equilibrium.

$A_{\mathcal{S}}7$. Let $\delta \in (0,1)$ be depreciation rate $:= \tilde{dep} = \delta \tilde{k}$.
As net investment = actual investment − break-even investment,

$$\begin{aligned} \Delta K &= I - \delta K \\ &\iff \Delta \tilde{K} = \tilde{I} - \delta \tilde{K} \\ &\implies \Delta \tilde{K} = 0 \iff \tilde{I} = \delta \tilde{K} \qquad \text{at the intersection.} \end{aligned}$$

Three possible outcomes:

i. If $\tilde{I} > \delta \tilde{K}, \Delta \tilde{K} > 0$ i.e., economic growth (per worker).
ii. If $\tilde{I} = \delta \tilde{K}, \Delta \tilde{K} = 0$ i.e., steady state.
iii. If $\tilde{I} < \delta \tilde{K}, \Delta \tilde{K} < 0$ i.e., economic decline (per worker).

Definition 2.1.1 (Steady state)**.** We arrive at *the* steady state, if

$$\Delta K = I - \delta K = 0.$$

In this state, the average standard of living is neither falling nor growing. I would however prefer growing $y = AP_L = (\frac{Y}{L})$.

Example. Japan pre vs. post WWII. Suppose the war kills a larger fraction of K than L. Then,

$$\begin{aligned} &\downarrow \tilde{K} \\ \implies &\downarrow \tilde{Y} && \text{since } MP_K > 0 \\ \implies &\downarrow \tilde{S} && \text{since } s > 0 \\ \implies &\downarrow \tilde{C} && \text{since } (1-s) > 0 \\ \implies &\downarrow \tilde{I} && \text{since } I = S \\ \implies &\downarrow \tilde{dep} && \text{since } \delta > 0 \end{aligned}$$

Remark. However, new steady state $:= \Delta K = 0 :=$ initial state.
Note nonetheless, $\tilde{K}_1 < I$, in the *very long-run*, dynamic path push the new LR equilibrium to the new (initial herein) steady state. That is, in the very long run, $\Delta \tilde{K} > 0$.

Example. Suppose the government wish to change the saving rate, e.g., $\uparrow s$. Then

$$\begin{aligned} &\uparrow s \\ \implies &\uparrow \tilde{S} && \text{since } s > 0 \\ \implies &\downarrow \tilde{C} && \text{since } 1 - s = MPC > 0 \\ \implies &\uparrow \tilde{I} && \text{since } I = S \end{aligned}$$

Remark. Note however, in new LR equilibrium,

$$\Delta\tilde{K} = \Delta\tilde{Y} = \Delta\tilde{dep} = 0.$$

That is, the same growth steady state.
In the *new steady state* through very long run dynamic path, as new $\Delta\tilde{K} > 0$,

$$\begin{aligned} &\uparrow s \\ \implies &\uparrow \tilde{S} && \text{since } s > 0 \\ \implies &\uparrow \tilde{C} \\ \implies &\uparrow \tilde{I} && \text{since } I = S \\ \implies &\uparrow \tilde{Y}, \tilde{K}, \tilde{dep}. \end{aligned}$$

$A_{\mathcal{S}}8.$ Now we assume

$$\frac{\Delta pop}{pop} = n = \frac{\Delta L}{L}.$$

It then follows that

$$\begin{aligned} \Delta K &= I - \delta K && \text{Aggregate} \\ \Delta\tilde{K} &= \tilde{I} - (n + \delta)\tilde{K} && \text{Per Worker} \end{aligned}$$

2.2 The Role of Technology Change

$A_{\mathcal{S}}9.$, Now, we allow *tech change.*

Definition 2.2.1 (Embodied Tech Change)**.** Let E be *labour biased technology.* Then, $L \times E :=$ the effective number of workers. By construction,

$$Y = F(K, L \times E),$$

such that $\uparrow E \implies$ tech improvement $\implies \uparrow MP_L \implies \uparrow (\frac{W}{P})$ and $\frac{\partial MP_K}{\partial E} = 0$.
Now, let's denote

$$\frac{\Delta E}{E} = g = \text{ the rate of labour augmenting tech progress.}$$

Note it follows that

$$\frac{\Delta(L \times E)}{L \times E} = \frac{\Delta L}{L} + \frac{\Delta E}{E} = n + g.$$

$A_{\mathcal{S}}10.$,Now we assume, different from $A_{\mathcal{S}}3.$, where we chose $\lambda = \frac{1}{L}$.
Instead, we choose $\lambda = \frac{1}{L \times E}$.
Then,

$$\frac{Y}{L \times E} = f(\frac{k}{L \times E}) = \tilde{\tilde{y}} = F(\tilde{\tilde{K}}).$$

Thus,

Aggregate	Per Worker	Per effective worker
$Y = F(K, L \times E)$	$\tilde{Y} = F(\tilde{K}, E)$	$\tilde{\tilde{Y}} = F(\tilde{\tilde{K}})$
$S = s \cdot Y$	$\tilde{S} = s \cdot \tilde{Y}$	$\tilde{\tilde{S}} = s \cdot \tilde{\tilde{Y}}$
$S = I$	$\tilde{S} = \tilde{I}$	$\tilde{\tilde{S}} = \tilde{\tilde{I}}$
$C = (1-s)Y$	$\tilde{C} = (1-s)\tilde{Y}$	$\tilde{\tilde{C}} = (1-s)\tilde{\tilde{Y}}$
$Y = C + I$	$\tilde{Y} = \tilde{C} + \tilde{I}$	$\tilde{\tilde{Y}} = \tilde{\tilde{C}} + \tilde{\tilde{I}}$
$Y = C + S$	$\tilde{Y} = \tilde{C} + \tilde{S}$	$\tilde{\tilde{Y}} = \tilde{\tilde{C}} + \tilde{\tilde{S}}$
$dep = \delta k$	$\tilde{dep} = \delta \tilde{k}$	$\tilde{\tilde{dep}} = \delta \tilde{\tilde{k}}$.

Definition 2.2.2 (Reconstructed break-even investment in very LR)**.**

$$\Delta K = I - \delta K \quad \text{Aggregate}$$
$$\Delta \tilde{K} = \tilde{I} - (n + \delta)\tilde{K} \quad \text{Per Worker}$$
$$\Delta \tilde{\tilde{K}} = \tilde{\tilde{I}} - (n + \delta + g)\tilde{\tilde{K}} \quad \text{Per Effective Worker}$$

Example. Suppose in the very long run $I = 0 = \tilde{I} = \tilde{\tilde{I}}$. Then,

$$\downarrow\downarrow\downarrow \tilde{\tilde{K}} = \left(\frac{\downarrow K}{\uparrow L \times \uparrow E} \right).$$

Thus,

$$\frac{\Delta \tilde{\tilde{K}}}{\tilde{\tilde{K}}} = -(n + \delta + g)$$
$$\implies \Delta \tilde{\tilde{K}} = -(n + \delta + g)\tilde{\tilde{K}}.$$

Proposition 2.2.2.1 (SS Growth Rates)**.**

	SS Growth Rate
$\tilde{\tilde{Y}} = \dfrac{Y}{L \times E}$	In SS, $\dfrac{\Delta\tilde{\tilde{Y}}}{\tilde{\tilde{Y}}} = 0$
$\tilde{Y} = \dfrac{Y}{L} = \tilde{\tilde{Y}} \times E$	In SS, $\dfrac{\Delta\tilde{Y}}{\tilde{Y}} = \dfrac{\Delta\tilde{\tilde{Y}}}{\tilde{\tilde{Y}}} + \dfrac{\Delta E}{E} = 0 + g = g > 0$
$Y = \tilde{\tilde{Y}} \times L \times E$	In SS, $\dfrac{\Delta Y}{Y} = \dfrac{\Delta\tilde{\tilde{Y}}}{\tilde{\tilde{Y}}} + \dfrac{\Delta L}{L} + \dfrac{\Delta E}{E} = 0 + n + g > 0$

Remark. This, as constructed based on exogenous growth theory, tells us the importance of g to the standard of living.

Theorem 2.2.2.2 (Golden Rule Steady State)**.** The GRSS is the SS that maximizes the level of $\tilde{\tilde{c}}$ in SS.

$$\max_{s} \tilde{\tilde{c}}^* = \max_{\tilde{\tilde{k}}^*} \tilde{\tilde{c}}^*$$

which occurs at, in SS,

$$MP_K = (n + \delta + g).$$

Development of the theory.

$$\begin{aligned}
&\tilde{\tilde{c}} = (1 - s)\tilde{\tilde{y}} \\
&\implies \tilde{\tilde{c}} = \tilde{\tilde{y}} - s\tilde{\tilde{y}} \\
&\implies \tilde{\tilde{c}} = F(\tilde{\tilde{K}}) - \tilde{\tilde{S}} && \text{by definition of } \tilde{\tilde{Y}} \text{ and } \tilde{\tilde{S}} \\
&\implies \tilde{\tilde{c}} = F(\tilde{\tilde{K}}) - \tilde{\tilde{I}} && \text{as } \tilde{\tilde{S}} = \tilde{\tilde{I}} \\
&\implies \tilde{\tilde{c}}^* = F(\tilde{\tilde{K}}^*) - \tilde{\tilde{I}}^* && \text{in steady state } * \\
&\implies \tilde{\tilde{c}}^* = F(\tilde{\tilde{K}}^*) - (n + \delta + g)\tilde{\tilde{K}}^* \\
&\implies \max_{\tilde{\tilde{K}}^*} \tilde{\tilde{c}}^* \iff \frac{d\tilde{\tilde{c}}^*}{d\tilde{\tilde{k}}^*} = 0 = MP_K - (n + \delta + g) && \text{as } \tilde{\tilde{c}}^* = F(\tilde{\tilde{K}}^*) - (n + \delta + g)\tilde{\tilde{K}}^* \\
&\iff MP_K = (n + \delta + g).
\end{aligned}$$

□

Corollary 2.2.2.2.1. It follows that,

$$S_{gold} = \alpha = \text{Capital's Income Share} = \frac{MP_K \cdot K}{Y}.$$

Proof. Since, in any SS,

$$\tilde{\tilde{I}} = (n + \delta + g)\tilde{\tilde{k}}.$$

In a GRSS,

$$MP_K = (n + \delta + g).$$

Thus,

$$\begin{aligned}
\tilde{\tilde{I}}^* = \tilde{\tilde{S}}^* = s_{gold}\tilde{\tilde{y}}^* = (n + \delta + g)\tilde{\tilde{K}}^* = MP_K \cdot \tilde{\tilde{K}}^* \\
\implies S_{gold} = \frac{MP_K\tilde{\tilde{K}}^*}{\tilde{\tilde{Y}}^*} = \frac{(\frac{R}{P})K^*}{Y^*} = \alpha.
\end{aligned}$$

□

3 Money, Prices, Inflation in the LR and very LR

Definition 3.0.1 (Money)**.** *Money* has three main functions:

1. Store of Values - transfer purchasing power to the future
2. Unit of Account - provides the terms in which prices and debts are recorded
3. Medium of Exchange - that which is used to by goods and services

Definition 3.0.2 (Money Demand)**.** We denote money demand as M^d. There are four types of money demand

1. Speculative - demand asset mix with some money to speculate on return
2. Precautionary - in case of emergency
3. Liquidity - improve liquidity to asset mix
4. Transactions - for day-to-day transactions.

Definition 3.0.3 (Monetary Policy Tools)**.** There are six tools:

1. Reserve ratios - of chartered banks (% of deposits held reserves)
2. Government deposits - moved around to size M^s
3. Official transactions in foreign assets - buying/selling foreign bonds
4. Open market operations - buying/selling of domestic assets, e.g., gov bonds
5. Bank rate (overnight interest rate) adjustment.

Definition 3.0.4 (Quantity Equation (transaction))**.**

$$MV = PT$$

where M is money supply, V is transactions velocity, P is price of an average transaction, T is the number of transactions.

Definition 3.0.5 (Quantity Equation (income))**.**

$$MV = PY$$

where M is money supply, V is income velocity, P is price of real output (GDP deflator), Y is real output (real GDP).

Consequences. Suppose we assume $\bar{V}$. Then

$$M^s = M = \frac{PY}{V} = M^d \implies \text{demand is not shifting.}$$

Furthermore, whoever controls M^s determines the size of nominal GDP in LR and very LR. Assume further that $\bar{Y}$. Then whoever controls money supply determines nominal price level in the LR. In the LR

$$\Delta M^S \implies \Delta P.$$

In the very LR, whoever picks the growth rate of money supply picks the inflation rate,

$$\begin{aligned}
&\frac{\Delta M^s}{M^s} + \frac{\Delta V}{V} = \frac{\Delta P}{P} + \frac{\Delta Y}{Y} \\
&\implies \pi = \frac{\Delta M^s}{M^s} + \frac{\Delta V}{V} - \frac{\Delta Y}{Y} \\
&\implies \frac{\Delta M^s}{M^s} = \frac{\Delta P}{P} = \pi \wedge \frac{\Delta V}{V} = \frac{\Delta Y}{Y} = 0.
\end{aligned}$$

Further, for expectations,

$$i = r_{EA} + \pi^e$$

where saving and investment determines r_{EA}, QTM determines π^e and fisher equation determines i. Thus,

$$\pi^e = \left(\frac{\Delta P}{P}\right)^e = \left(\frac{\Delta M^s}{M^s}\right)^e + \left(\frac{\Delta V}{V}\right)^e - \left(\frac{\Delta Y}{Y}\right)^e.$$

Remark. The reputation of the central bank influences the determination of inflationary expectations in LR and very LR (theyby influences the setting of i).

$$\Delta i = \Delta \Pi^e.$$

Suppose there is a surprise inflation,

$$\begin{aligned}
&\Pi = \Pi^e \iff r = r^e \\
&\Pi > \Pi^e \iff r < r^e \qquad \text{borrows pay less than what lenders expected in real terms} \\
&\Pi < \Pi^e \iff r > r^e
\end{aligned}$$

4 Unemployment in the LR

Definition 4.0.1 (Working Age Population - WAP)**.** WAP is the entire population between the ages of 15 and death.

Definition 4.0.2 (Employed - E)**.** Employed (E) if work full-time or part-time in a *paid* job (on sick leave, vacation, on strike).
Remark. In some sense an overestimation as it overlooks underemployment and discouraged workers (labour force dropout; not unemployed).

Definition 4.0.3 (Underemployment)**.** The population with part-time jobs who want full-time jobs.

Definition 4.0.4 (Unemployed - U)**.** A person aged 15 and up is considered unemployed (U) if they

1. are without a paid job
2. sought paid work actively
3. available for work.

Definition 4.0.5 (Labour Force - LF)**.** Population employed plus unemployed

$$LF = U + E.$$

Remark.

$$WAP = LF \cup \neg LF.$$

Definition 4.0.6 (Unemployment Rate - UR)**.** proportion of unemployed population to labour force

$$\begin{aligned} UR &= \frac{U}{LF} \\ &= \frac{LF - E}{LE} \\ &= 1 - \frac{E}{LF} \\ &= \frac{U}{U + E}. \end{aligned}$$

Definition 4.0.7 (Employment Rate - ER)**.**

$$ER = E/WAP = E/(LF + NILF) = E/(E + U + NILF).$$

Definition 4.0.8 (Participation Rate - PR)**.**

$$PR = LF/WAP = LF/(KF + NILF) = ER/(1 - UR)$$

Proposition 4.0.8.1 (Causes of Unemployment)**.** There are two basic causes of U,

1. short-run *cyclical unemployment* denoted U_{cyc}
2. long-run *natural rate level of unemployment*, denoted $\bar{U}$ = Frictional + Structural + Seasonal U.

Thus, total measure unemployment

$$U_{total} = SRU + LRU = U_{cyc} + \bar{U}.$$

Further in LR

$$\bar{Y} = \bar{U} \implies \text{suppose} Y < \bar{Y} \iff U > \bar{U} \iff U_{cyc} > 0.$$

To counter this we need expansionary fiscal policy ($\uparrow T, tr, G.$)

Derivation of the UR in LR. Assume

1. LF is constant
2. A constant fraction of the employed lose their jobs (S)
3. A constant fraction of the unemployed find a job (F)

Then,

$$\begin{aligned} \bar{LF} = \bar{E} + \bar{U} &\implies E = LF - U \\ \bar{UR} = \frac{\bar{U}}{LF} &\implies sE = FU \\ &\implies s(LF - U) = FU \\ &\implies s(1 - \frac{U}{LF}) = F(\frac{u}{LF}) \\ &\implies 0 < \bar{UR} = \frac{S}{S + F} < 1. \end{aligned}$$

Suppose we change EI to be more generous $\uparrow S$ and $\downarrow F \implies \uparrow UR$

5 The open Economy in LR

Model Assumptions.
We adhere to all assumptions in Ch. 3 and replace closed economy with open economy, in addition

i. Small Open Economy (SOE) – an economy so small that it cannot affect world markets. That is, we define r_w to be exogenous: at equilibrium

$$S_w = I_w \iff r_w = r^*.$$

ii. Perfect Financial Capital Mobility – an immediate consequence is that financial capital is free to flow into, or out of the country.

iii. No Risk Premium ($\theta = 0$). If a country A is more risky then B, then $\theta_A < \theta_B$. The assumptions imply that the economy concerned is *as risky as* the rest of the world:

$$r = r_w + \theta \implies r = r_w.$$

Thus, domestic and foreign financial assets are *perfect substitutes* in LR equilibrium. In particular, the world real rate of interest and the domestic rate are perfectly correlated:

$$\Delta r_w \text{ causes } \Delta r.$$

Proposition 5.0.0.1 ($r = r_w$)**.** $r = r_w$ not only at some point, but as soon as possible.

Proof. We are given that $r = r_w + \theta$. By assumption iii., we know that $r = r_w$ *at some points.* By assumption i., we know that r_w is given. By assumption ii., we know that we achieve $r = r_w$ as soon as possible. ☐

Example. Consider $r > r_W$. Then,

$$\begin{aligned} r > r_w &\implies \text{Better fin invest in domestic economy} \\ &\implies \text{Saver decides to buy more domestic fin assets} \\ &\implies \uparrow \text{inflow of fin capital} \\ &\implies \uparrow P \text{ of dom fin assets} \\ &\implies \downarrow r \\ &\implies r = r_w. \end{aligned}$$

Definition 5.0.1 (Real domestic output or income)**.**

$$Y = C + I + G + NX,$$

where NX = net exports $= Y - (C + I + G) = (Y - C - G) - I = S_{NAT} - I$.
That is,

$$NX = S_{NAT} - I = \text{domestic saving - domestic inv.}$$

In short,

$$\begin{aligned} NX &= S_{NAT} - I \\ \iff S_{NAT} &= I + NX \\ \iff I &= S_{NAT} - NX. \end{aligned}$$

Definition 5.0.2 (Saving from rest of the world)**.** We define net saving in the domestic economy coming from foreign sources as

$$S_{ROW} = -NX.$$

Thus,

$$I = S_{NAT} - NX \implies I = S_{NAT} + S_{ROW}.$$

Intuitively, domestic investment equals domestic savings from *all sources.*
Example. Suppose $NX = 0$. Then, as $NX = (Y - C - G) - I$, $S_{NAT} = I$, *i.e.*, all income is spent (none saved). In such a state, *net*, we are neither borrowing nor lending with the rest of the world, i.e., $S_{ROW} = 0 \iff S_{NAT} = I$.
Suppose $NX > 0$. Then, $S_{NAT} > I$. As $I = S_{NAT} + S_{ROW}, S_{ROW} < 0$. Here we have net inflow of *IOUs* and net outflow of financial capital to the ROW. That is, we are *lending* to the rest of the world.
Suppose $NX < 0$, then $S_{ROW} > 0$. That is, we are borrowing from the ROW. Thus, we have *outflow* of *IOUs* and *inflow* of financial capital from the ROW.

Definition 5.0.3 (Nominal Exchange Rate)**.** *Nominal Exchange Rate* is the relative price of the nominal currency of two countries, i.e., number of foreign currency units per domestic unit.

$$e = E_{nominal} \wedge e' = \frac{1}{E_{nominal}}.$$

Note: it consists of both the domestic and foreign inflation as distortion sources.

Definition 5.0.4 (Real Exchange Rate)**.** The *real exchange rate* is the *relative* price of the real output of two countries, i.e., the number of foreign units of *real output* per domestic unit of *real output.*

$$\varepsilon = \frac{e \cdot P}{P_f} = \frac{\$_{foreign} \text{ cost of 1 unit domestic real output}}{\$_{foreign} \text{ cost of 1 unit of foreign real output.}}$$

Definition 5.0.5 (Relative PPP)**.** Relative Purchasing Power Parity states that in the long run:

$$\varepsilon = \left(\frac{e \cdot P}{P_f}\right) = t \text{ s.t. } t \in \mathbb{R}^+.$$

Theory Assumptions

1. LR
2. sufficient openness for trade and capital flows
3. Govs don't exert much control over the price level and/or the nominal exchange rate, i.e., flexible ε.

Definition 5.0.6 (Absolute PPP)**.** Absolute Purchasing Power Parity states that in the long run:

$$\varepsilon = \left(\frac{e \cdot P}{P_f}\right) = 1.$$

Theory Assumption

1. LR
2. sufficient openness for trade and capital flows
3. Govs don't exert much control over the price level and/or the nominal exchange rate, i.e., flexible ε.
4. Identical outputs (perfect substitutes)
5. *all* trades and services are tradable internationally
6. *no* transportation costs
7. *no* transaction costs, e.g., tariffs, etc.

Theorem 5.0.6.1 (PPP)**.** Absolute PPP implies Relative PPP. If the real exchange rate is constant, then

$$\begin{aligned} \left(\frac{\Delta\varepsilon}{\varepsilon}\right) &= 0 = \left(\frac{\Delta e}{e}\right) + \left(\frac{\Delta P}{P}\right) - \left(\frac{\Delta P_f}{P_f}\right) \\ \iff \left(\frac{\Delta e}{e}\right) &= \left(\frac{\Delta P_f}{P_f}\right) - \left(\frac{\Delta P}{P}\right) = \pi_f - \pi \end{aligned}$$

Proof. Trivial, as Absolute PPP $\subseteq$ Relative PPP. □

Theorem 5.0.6.2 (Three Transformation)**.**

1. Convert Nominal Variables into Real terms (remove inflationary distortion).
2. Convert Aggregate Variables into real per capita terms (remove population distortion).
3. Convert International Variables into real PPP-adjusted terms (remove purchasing power distortion).

Definition 5.0.7 (Net Export Function)**.** We assume that the net exports can be explained by a relationship:

$$NX(\varepsilon, Y) = Ex(\varepsilon) - Im(\varepsilon, Y) = NX_0 - NX_1 \cdot \varepsilon.$$

Where we assume that

1. $\frac{\partial Ex}{\partial \varepsilon} < 0$; thus
$$\frac{\partial NX}{\partial \varepsilon} < 0.$$
2. $\frac{\partial Im}{\partial \varepsilon} > 0$
3. $\frac{\partial Im}{\partial Y} > 0$; thus
$$\frac{\partial NX}{\partial Y} < 0.$$

Definition 5.0.8 (Goods Market Equilibrium)**.**

$$Y = C + I + G + NX.$$

$$\begin{array}{c|l} Y = A \cdot F(K, L) & \text{Prod. Function} \\ C = C_0 + C_1 \cdot (Y - T) & \text{Cons. function} \\ I = I_0 - I_1 \cdot r & \text{Inv. Function} \\ G = G_0 \wedge T = T_0 & \text{Exog., fiscal policy} \\ r = r_w & \text{Exog., real interest rate} \\ NX(\varepsilon, Y) = Ex(\varepsilon) - Im(\varepsilon, Y) = NX_0 - NX_1 \cdot \varepsilon & \text{NX function} \end{array}$$

In which we can derive ε at

$$S_{NAT} - I = NX \implies \varepsilon^*$$

6 Shocks and IS-LM Model

Definition 6.0.1 (IS Curve)**.** The IS Curve plots *all* the (y, r) points such that $y^s = y^d$, i.e., goods market equilibrium. It is assumed to be in a closed Economy. When, $y^s = y^d, S_{NAT} = I$. The IS curve plots all the (y, r) s.t. $I = S$, i.e., Loanable Funds Market equilibrium:

$$y = \left(\frac{C_0 + I_0 + G_0 - C_1 T_0}{1 - C_1} \right) - \frac{I_1}{1 - C_1} r.$$

Proof. Model Derivations.

$$\begin{aligned}
&y = C + I + G \\
&\iff y = (C_0 + C_1(y - T)) + (I_0 - I_1 \cdot r) + G_0 && \text{def of } C, I, G \\
&\iff (1 - C_1)y = [C_0 - C_1 T + I_0 + G_0] - I_1 \cdot r \\
&\iff y = \frac{1}{1 - C_1} (C_0 - C_1 T + I_0 + G_0) - \frac{I_1}{1 - C_1} r \\
&\iff y = \left(\frac{C_0 + I_0 + G_0 - C_1 T_0}{1 - C_1} \right) - \frac{I_1}{1 - C_1} r && \text{as needed.}
\end{aligned}$$

Where,

$$\frac{\partial r}{\partial y} = \frac{1}{\frac{\partial y}{\partial r}} = \frac{1}{-\frac{I_1}{1 - C_1}} = -\frac{1 - C_1}{I_1} < 0.$$

From which it follows that

$$\uparrow\downarrow (C_0 \vee I_0 \vee G_0) \text{ and } \downarrow\uparrow T_0 \text{ shift IS curve to the right/left.}$$

Note that *fiscal policy* can shift the IS, and consumer/business confidence can shift the IS. □

Definition 6.0.2 (LM curve)**.** Analogously, the LM curve plots all the (y, r) s.t. $M^s = M^d$. Intuitively, it is a curve of all the possible money market equilibria, i.e., the liquidity market equilibrium curve:

$$y = \left(\frac{1}{k}\right)\left(\frac{M}{P}\right) + \left(\frac{h}{k}\right) r.$$

Where k refers to the income sensitivity of money demand and h refers to the interest sensitivity of money demand.

Proof. Model Derivations.

Nominal	Real
$M^s = M^d$	$\frac{M^s}{P} = \frac{M^d}{P}$
$M = P \times (k \cdot y - h \cdot r)$	$\frac{M}{P} = (k \cdot y - h \cdot r)$
	$y = \left(\frac{1}{k}\right)\left(\frac{M}{P}\right) + \left(\frac{h}{k}\right) r$
	$\frac{\partial r}{\partial y} = \frac{k}{h} > 0$

Where, (1) central banks determines M.
(2) $P = GDP_{deflator} = P_{nominal}$
In SR, P is *fixed.*
In LR, P is perfectly *flexible.*
In SR,

$$\frac{M^d}{P} = \frac{M^s}{P} = \text{real money supply.}$$

Where

$$M^d = P(k \cdot y - h \cdot r) \qquad \text{nom money demand}$$

$$\frac{M^d}{P} = (k \cdot y - h \cdot r) \qquad \text{real money demand}$$

Observe that

$$\frac{\partial(\frac{M^d}{P})}{\partial y} = k > 0 \quad \text{and} \quad \frac{\partial(\frac{M^d}{P})}{\partial r} = -h < 0.$$

□

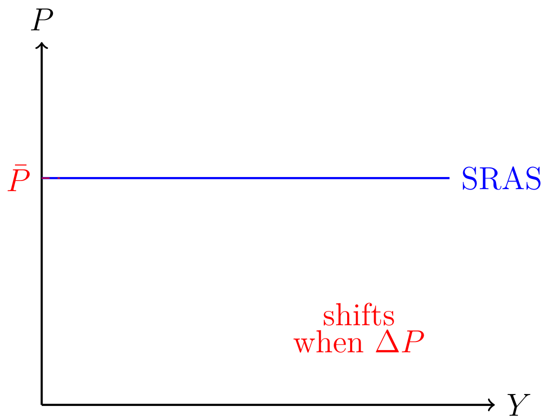
7 AS-AD

Remarks. AS-AD model is equivalent to IS-LM model in that they rely on a same set of assumptions.

Definition 7.0.1 (Model Assumption)**.**

1. AS/Ad Model has the *same* list of assumptions used in the IS/LM Model.
2. Therefore, the AS/AD Model is the *same* as the IS/LM model
3. We must get the *same* answer to questions with either model
4. two models complement each other.

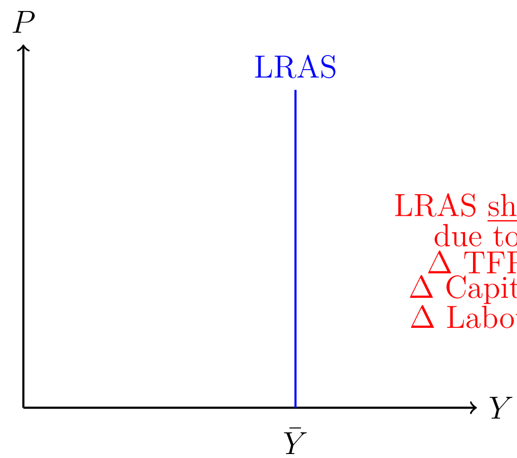
Definition 7.0.2 (Short-Run AS)**.** The SRAS curve is *perfectly flat,* due to the model assumption that P is *fixed* in the SR. In the SR, suppliers are willing to supply at *any* level of out put at $P = \overline{P}$.



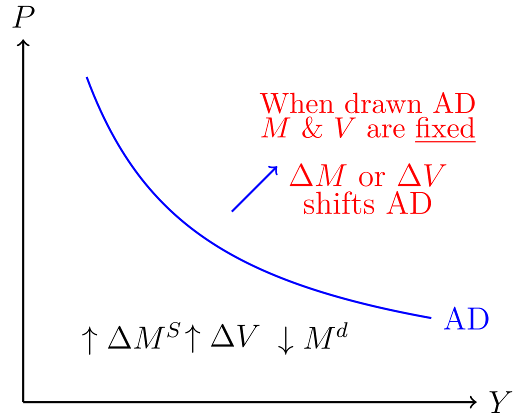
Definition 7.0.3 (The Long-Run Aggregate Supply Curve)**.** THE LRAS is *perfectly vertial* due to

1. Capital is *fixed* and fully employed
2. Labour is *fixed* and fully employed
3. Tech is *fixed*
4. P is *perfectly flexible* in the LR.

$$y^s = \overline{y} = A \cdot F(\overline{K}, \overline{L}).$$



Definition 7.0.4 (The Aggregate Demand Curve)**.** The AD curve plots all the levels of real aggregate output demanded at each aggregate price level. It is downward sloping and nonlinear.



Theory Derivations. Recall that by assumption M and V are fixed

$$\overline{M} \cdot \overline{V} = P \cdot Y.$$

Thus,

$$\uparrow P \iff \downarrow Y \quad \downarrow P \iff \uparrow Y,$$

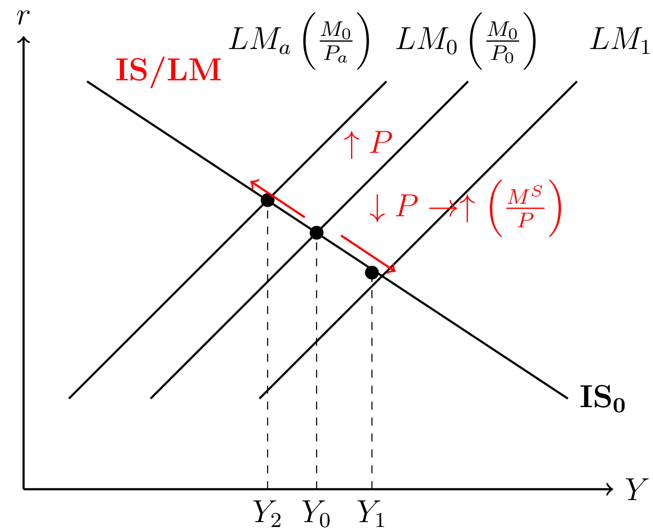
that is, downward sloping. **On Shocks.**
Note that

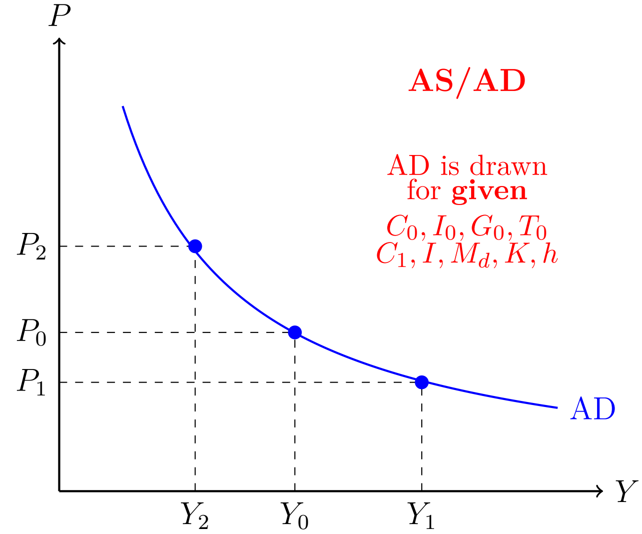
$$M^s = M = \frac{P \cdot Y}{V} = M^d \text{ and } M^d = P(ky - hr).$$

Thus, note the asymmetry

$$\downarrow M^d \iff \uparrow V \implies \uparrow AD \text{ and } \uparrow M^s \implies \uparrow AD.$$

□


r
$LM_a\left(\frac{M_0}{P_a}\right)$
$LM_0\left(\frac{M_0}{P_0}\right)$
$LM_1\left(\frac{M_0}{P_1}\right)$
IS/LM
$\uparrow P$
$\downarrow P \rightarrow \uparrow \left(\frac{M^S}{P}\right)$
IS_0
Y
Y_2 Y_0 Y_1


P
AS/AD
AD is drawn
for **given**
C_0, I_0, G_0, T_0
C_1, I, M_d, K, h
P_2
P_0
P_1
AD
Y
Y_2 Y_0 Y_1

8 Mundell-Fleming Model

Definition 8.0.1 (Mundell-Fleming Model)**.** Modified IS/LM model so that it applies to a small open economy.

1. We retain all assumptions except closed assumption
2. We assume SOE
3. Assume perfect financial capital/mobility
4. No risk premium ($\theta = 0$)
$$r = r_w^*.$$
5. Assume in LR
$$NX - NX_0 - NX_1 \cdot \varepsilon \iff \varepsilon = \frac{e \cdot P_d}{P_f}$$
6. In SR
$$\varepsilon = \frac{e \cdot \overline{P_d}}{\overline{P_f}} \iff NX = NX_0 - NX_1 \left(\frac{\overline{P_d}}{\overline{P_f}} \right) \cdot e \iff NX = NX_0 - \tilde{NX_1} \cdot e.$$
In SR, Δe *casues* $\Delta \varepsilon$, which in turns causes ΔNX.

Definition 8.0.2 (Fixed e rate regime)**.**

$$e = e_f$$

is *exog or fixed* thus, in SR
$$\varepsilon = \overline{\varepsilon} \text{ and } NX = \overline{NX}.$$

Definition 8.0.3 (Flexible e rate regime)**.** e is *endog* in SR

$$\varepsilon \text{ and } NX \text{ are endog in SR.}$$

Definition 8.0.4 (Model with Flexible Exchange Rates).

$$Y = C + I + G + NX \text{ s.t. } NX = NX_0 - \tilde{NX}_1 \cdot e.$$

Money Market Assumptions	
<u>Supply Side</u>	
$M^S = M =$ nominal money supply	(set by CB, exogenous)
$P =$ GDP deflator	(fixed in SR)
$\left(\frac{M^S}{P}\right) = \left(\frac{M}{P}\right)$	Real money supply (fixed in SR)
<u>Demand Side</u>	
$M^d = P \cdot (k \cdot Y - h \cdot r)$	Nominal money demand
$\left(\frac{M^d}{P}\right) = (k \cdot Y - h \cdot r)$	Real money demand
<u>Equilibrium</u>	
$M^S = M^d$	or $\left(\frac{M^S}{P}\right) = \left(\frac{M^d}{P}\right)$

Remarks. Monetary policy in flexible e regime is enhanced whereas fiscal policies are mitigated.

Definition 8.0.5 (Model with Fixed Exchange Rates).

The CB picks a desired/target nominal e rate, and they promise to do whatever is needed to make $e = e_f$ occur.

Exchange Rate Regime	**Endog Vars**	**IS Shocks**	**LM Shocks**	$\uparrow r^*$
Flexible e	Y, e	$\Delta Y^*_{\text{SR}} = 0$	Max ΔY^*_{SR}	$\uparrow Y^*_{\text{SR}}$
Fixed e	Y, M^S	Max ΔY^*_{SR}	$\Delta Y^*_{\text{SR}} = 0$	$\downarrow Y^*_{\text{SR}}$

A tool-kit of monetary operations:

Government Deposits

- *Expansionary:* Shift deposits to commercial banks $\Rightarrow\uparrow$ reserves $\Rightarrow\uparrow M^s$
- *Contractionary:* Shift deposits to central bank $\Rightarrow\downarrow$ reserves $\Rightarrow\downarrow M^s$

Official Transactions in Foreign Assets

- *Expansionary:* Buy foreign bonds $\Rightarrow$ inject domestic currency $\Rightarrow\uparrow M^s$
- *Contractionary:* Sell foreign bonds $\Rightarrow$ withdraw domestic currency $\Rightarrow\downarrow M^s$

Open Market Operations (Domestic Bonds)

- *Expansionary:* Buy government bonds $\Rightarrow\uparrow$ bank reserves $\Rightarrow\uparrow M^s$
- *Contractionary:* Sell government bonds $\Rightarrow\downarrow$ bank reserves $\Rightarrow\downarrow M^s$

Bank Rate (Overnight Interest Rate)

- *Expansionary:* Lower bank rate $\Rightarrow$ cheaper borrowing for banks $\Rightarrow\uparrow M^s$
- *Contractionary:* Raise bank rate $\Rightarrow$ more expensive borrowing $\Rightarrow\downarrow M^s$