MGEC06
Intermediate Macroeconomics II

Lecture Notes

documented by Eric Wu

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Textbook: Gregory Mankiw and William Scarth, Macroeconomics 5nd Canadian Edition, Worth Publishing, 2015.

1 Business Cycle Theory

1.1 Shocks and IS-LM Model

Definition 1.1.1 (IS Curve)**.** The IS Curve plots *all* the (y, r) points such that $y^s = y^d$, i.e., goods market equilibrium. It is assumed to be in a closed Economy. When, $y^s = y^d, S_{NAT} = I$. The IS curve plots all the (y, r) s.t. $I = S$, i.e., Loanable Funds Market equilibrium:

$$y = \left(\frac{C_0 + I_0 + G_0 - C_1 T_0}{1 - C_1}\right) - \frac{I_1}{1 - C_1} r.$$

Proof. Model Derivations.

$$\begin{aligned}
& y = C + I + G \\
\iff & y = (C_0 + C_1(y - T)) + (I_0 - I_1 \cdot r) + G_0 && \text{def of } C, I, G \\
\iff & (1 - C_1)y = [C_0 - C_1 T + I_0 + G_0] - I_1 \cdot r \\
\iff & y = \frac{1}{1 - C_1}(C_0 - C_1 T + I_0 + G_0) - \frac{I_1}{1 - C_1} r \\
\iff & y = \left(\frac{C_0 + I_0 + G_0 - C_1 T_0}{1 - C_1}\right) - \frac{I_1}{1 - C_1} r && \text{as needed.}
\end{aligned}$$

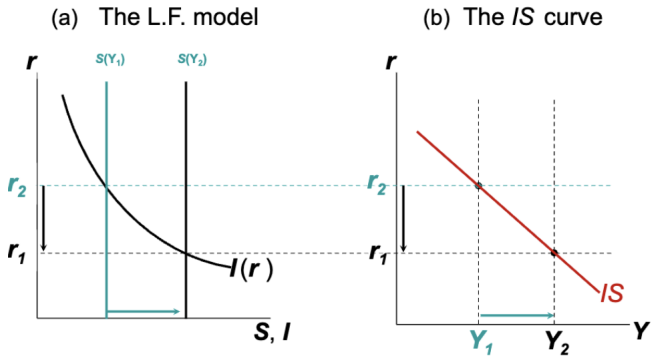
Where,

$$\frac{\partial r}{\partial y} = \frac{1}{\frac{\partial y}{\partial r}} = \frac{1}{-\frac{I_1}{1 - C_1}} = -\frac{1 - C_1}{I_1} < 0.$$

From which it follows that

$$\uparrow\downarrow (C_0 \vee I_0 \vee G_0) \text{ and } \downarrow\uparrow T_0 \text{ shift IS curve to the right/left.}$$

Note that *fiscal policy* can shift the IS, and consumer/business confidence can shift the IS. □



Definition 1.1.2 (LM curve)**.** Analogously, the LM curve plots all the (y, r) s.t. $M^s = M^d$. Intuitively, it is a curve of all the possible money market equilibria, i.e., the liquidity market equilibrium curve:

$$y = \left(\frac{1}{k}\right)\left(\frac{M}{P}\right) + \left(\frac{h}{k}\right) r.$$

Where k refers to the income sensitivity of money demand and h refers to the interest sensitivity of money demand.

Proof. Model Derivations.

Nominal	Real
$M^s = M^d$	$\frac{M^s}{P} = \frac{M^d}{P}$
$M = P \times (k \cdot y - h \cdot r)$	$\frac{M}{P} = (k \cdot y - h \cdot r)$
	$y = \left(\frac{1}{k}\right)\left(\frac{M}{P}\right) + \left(\frac{h}{k}\right) r$
	$\frac{\partial r}{\partial y} = \frac{k}{h} > 0$

Where, (1) central banks determines M.
(2) $P = GDP_{deflator} = P_{nominal}$
In SR, P is *fixed.*
In LR, P is perfectly *flexible.*
In SR,

$$\frac{M^d}{P} = \frac{M^s}{P} = \text{real money supply.}$$

Where

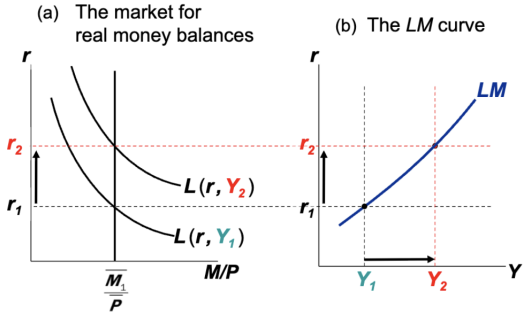
$$M^d = P(k \cdot y - h \cdot r) \qquad \text{nom money demand}$$

$$\frac{M^d}{P} = (k \cdot y - h \cdot r) \qquad \text{real money demand}$$

Observe that

$$\frac{\partial(\frac{M^d}{P})}{\partial y} = k > 0 \quad \text{and} \quad \frac{\partial(\frac{M^d}{P})}{\partial r} = -h < 0.$$

□



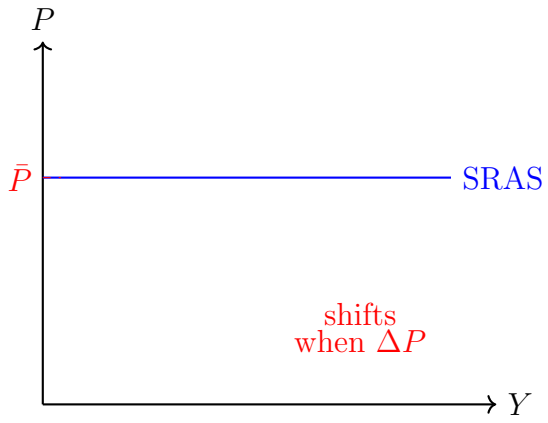
1.2 AS-AD

Remarks. AS-AD model is equivalent to IS-LM model in that they rely on a same set of assumptions.

Definition 1.2.1 (Model Assumption)**.**

1. AS/Ad Model has the *same* list of assumptions used in the IS/LM Model.
2. Therefore, the AS/AD Model is the *same* as the IS/LM model
3. We must get the *same* answer to questions with either model
4. two models complement each other.

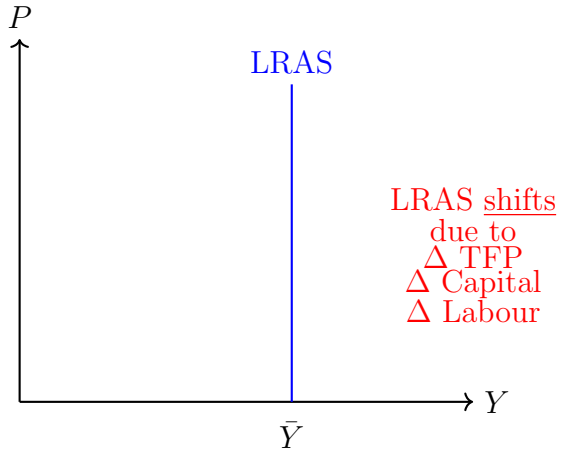
Definition 1.2.2 (Short-Run AS)**.** The SRAS curve is *perfectly flat,* due to the model assumption that P is *fixed* in the SR. In the SR, suppliers are willing to supply at *any* level of out put at $P = \overline{P}$.



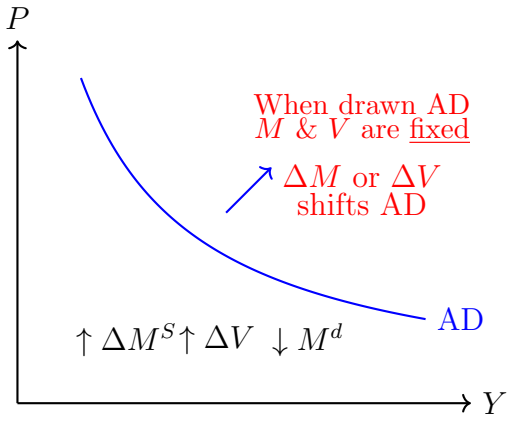
Definition 1.2.3 (The Long-Run Aggregate Supply Curve)**.** THE LRAS is *perfectly vertial* due to

1. Capital is *fixed* and fully employed
2. Labour is *fixed* and fully employed
3. Tech is *fixed*
4. P is *perfectly flexible* in the LR.

$$y^s = \overline{y} = A \cdot F(\overline{K}, \overline{L}).$$



Definition 1.2.4 (The Aggregate Demand Curve)**.** The AD curve plots all the levels of real aggregate output demanded at each aggregate price level. It is downward sloping and nonlinear.



Theory Derivations. Recall that by assumption M and V are fixed

$$\overline{M} \cdot \overline{V} = P \cdot Y.$$

Thus,

$$\uparrow P \iff \downarrow Y \quad \downarrow P \iff \uparrow Y,$$

that is, downward sloping. **On Shocks.**
Note that

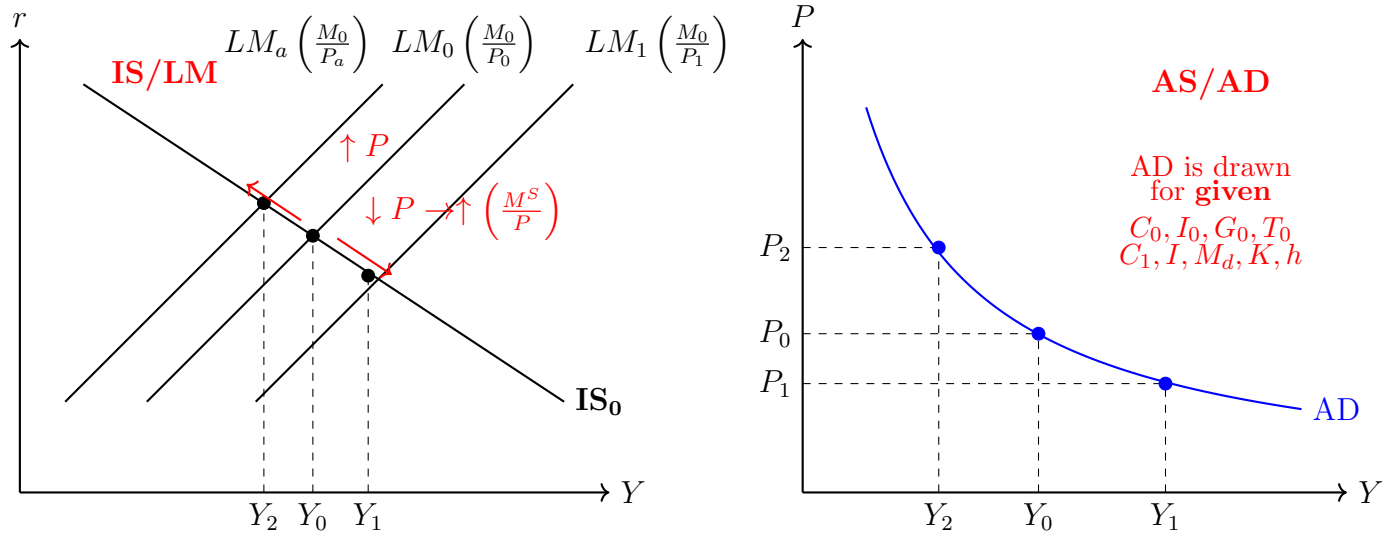
$$M^s = M = \frac{P \cdot Y}{V} = M^d \text{ and } M^d = P(ky - hr).$$

Thus, note the asymmetry

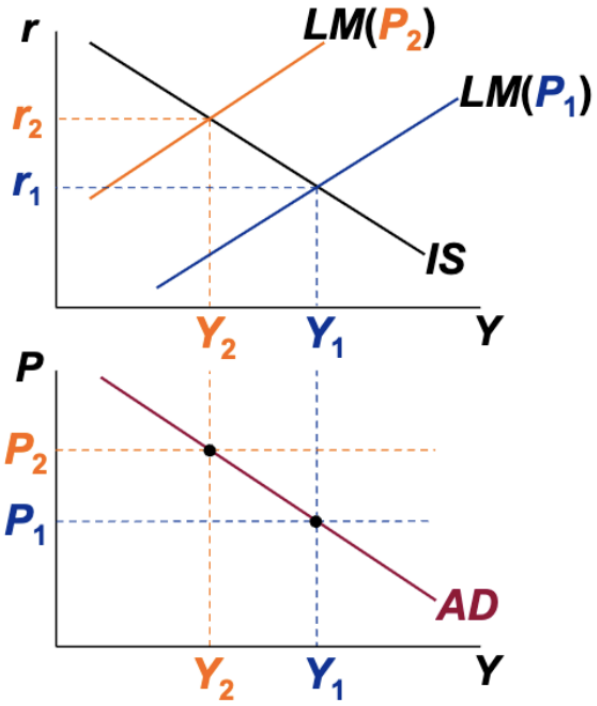
$$\downarrow M^d \iff \uparrow V \implies \uparrow AD \text{ and } \uparrow M^s \implies \uparrow AD.$$

□

Graphically,

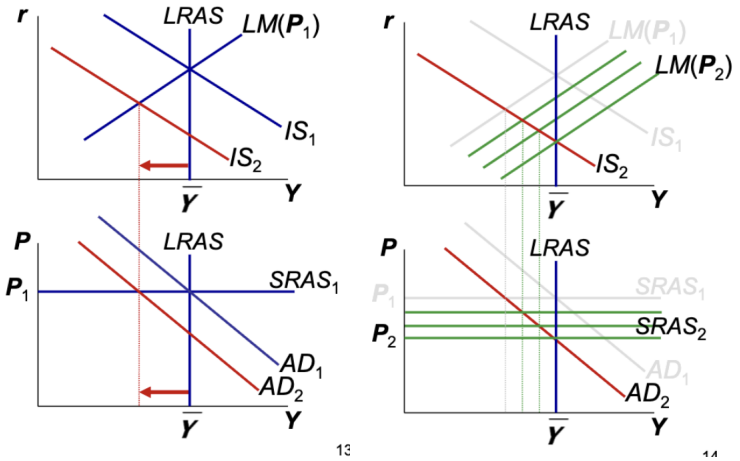


Also, the AD is derived:



2 Aggregate Supply Models

Remarks 2.0.0.0.1. In the real world, SRAS is upward rising: in contradistinction, in Keysian framework, P is assumed to be completely constant (living in depression) in short run and perfectly flexible in long run; in classical framework, expensionary fiscal shocks lead to lower prices, thereby leading to LM shock to the right keeping the output constant: the only change occurs to price level (Invisible Hand).
Graphically, on the left the graph illustrate Keynesian effect; on the right classical effect. New Keynesian attempts to introduce partial rigidity to previous models.



Remarks 2.0.0.0.2 (Model Implication)**.** All three models herein imply

$$Y = \overline{Y} + \alpha \left(P - P^e \right)$$

where Y is the *aggregate output*; $\overline{Y}$ is the *natural rate of output*, α is a positive parameter for the the difference between P the *actual price level* and P^e the *expected price level.*

2.1 The Sticky-Price Model

Remarks 2.1.0.0.1 (Assumptions)**.** In Sticky-Price Model we assume *firms set their own prices as in monopolistic competition.* There are three reasons for sticky prices:

1. long-term contracts between firms and customers
2. menu costs
3. firms not wishing to annoy customers with frequent price changes

Definition 2.1.1 (Individual Firms' Desired Price)**.** We define *Individual Firms' Desired Price* for flexible prices such that

$$p_1 = P + a\left(Y - \overline{Y}\right).$$

Where $a > 0$ Note that we express these in ln . Suppose we have two types of firms, (1) firms with flexible prices; (2) firms with sticky prices. For type (2) firms, they must set their price before they know P and Y and so their desire price is

$$p_2 = P^e + a(Y^e - \overline{Y}^e).$$

Suppose that sticky price firms expect $Y^e = \overline{Y}^e$. Then,

$$p_1 = P^e.$$

Let s denote the fraction of firms with sticky prices. Then, we can write the overall (combining flexible and sticky price firms) price as

$$P_{aggregate} = sp_2 + (1-s)p_1 = sP^e + (1-s)\left[P + a(Y - \overline{Y})\right]$$

From which it follows that

$$\begin{aligned} sP &= sP^e + (1-s)\left[a(Y - \overline{Y})\right] \\ &\implies P = P^e + \left[\frac{(1-s)a}{s}\right]\left(Y - \overline{Y}\right) \\ &\implies Y = \overline{Y} + \left[\frac{s}{(1-s)a}\right](P - P^e) \\ &= \overline{Y} + \alpha(P - P^e) \qquad \text{where } \alpha = \left[\tfrac{s}{(1-s)a}\right] \end{aligned}$$

Observe that when $s = 0$, i.e., there is no sticky price firms, we have

$$Y = \overline{Y},$$

that is, the classical framework. Suppose $s = 1$, i.e., every firms is sticky price firms, then

$$P = P^e$$

that is, the Keynesian framework – as price is now fully exogenous.
Lastly, the smaller is s, the bigger is the effect of ΔY on P.

Proposition 2.1.1.1 (Shocks)**.** In this model, change in P^e and $\overline{Y}$ will cause shifts of SRAS. For example, suppose $\overline{Y} \uparrow$, then P, i.e., supply shift to the right. Suppose $P^e \uparrow$. Then P shift to the left.

2.2 Sticky-Wage Model

Remarks 2.2.0.0.1 (Assumptions)**.** We assume that firms and workers negotiate contracts and fix the nominal wage before they know what the price level will turn out (usually through a labor union). The set of nominal wages is the product of a target real wage and the expected price level:

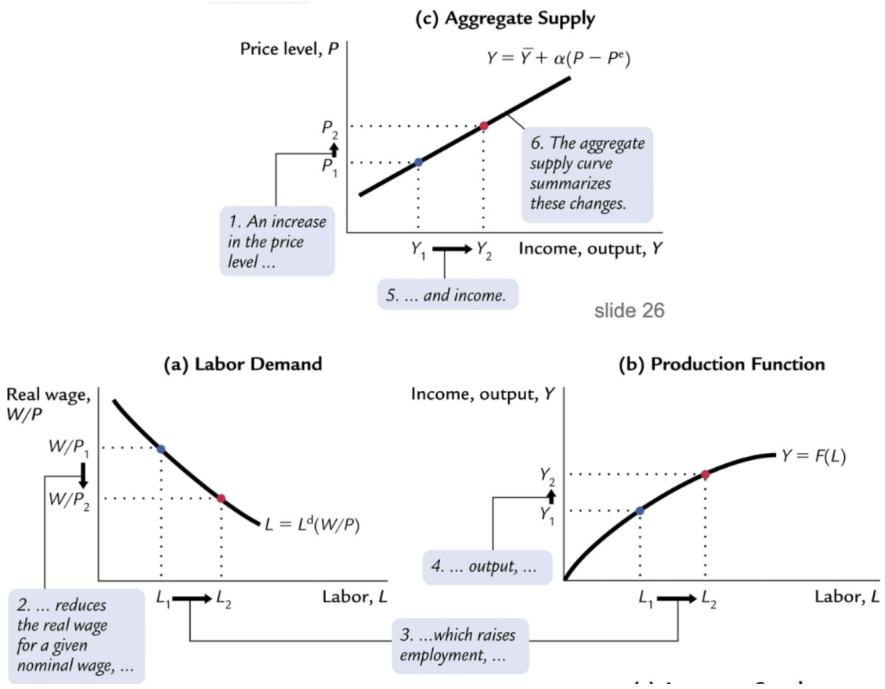
$$W_{nominal} = \omega \times Pe$$

where ω is the target real wage . And so

$$\frac{W}{P} = \omega \times \frac{P^e}{P},$$

where $\frac{W}{P}$ is the *real wage*, and $\frac{P^e}{P}$ is *expected price level relative to actual price level*.

$$\begin{array}{ll} \text{If } P^e = P, & \text{we are in equilibrium.} \\ \text{If } P^e > P, & \Rightarrow \text{real wage too high} \Rightarrow L \downarrow, Y \downarrow . \\ \text{If } P^e < P, & \Rightarrow \text{real wage too low} \Rightarrow L \uparrow, Y \uparrow . \end{array}$$



Proposition 2.2.0.1 (Two Problems of Sticky Wage Model)**.**

1. *Labour Market Does Not Clear.* Labour supply is assumed to follow labour demand.
2. *Real Wage,* $\frac{W}{P}$*, is Counter-Cyclical.*

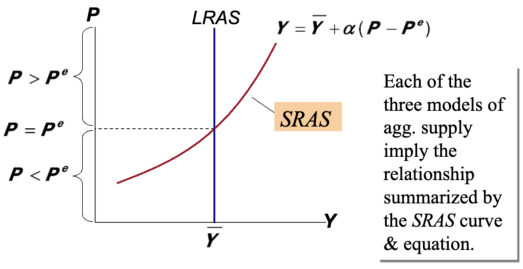
2.3 Imperfect-Information Model

Remarks 2.3.0.0.1 (Assumptions)**.**

1. All wages and prices are perfectly flexible, all markets are clear.
2. Each supplier produces one good, consumes many goods.
3. Each supplier knows the nominal price of the good she produces, but does not know the overall price level.

Proposition 2.3.0.1 (Stickiness)**.**

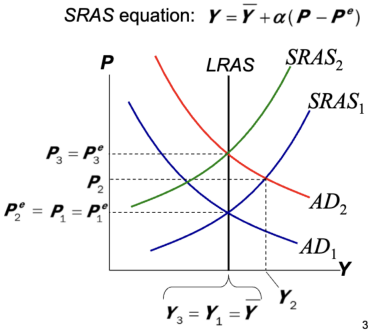
1. Supply of each good depends on its relative price: $\frac{P_{nominal}}{P_{aggregate}}$
2. Supplier does not know price level at the time she makes her production decision, so uses the expected price level, P^e.
3. Suppose $P \uparrow$ while P^e does not.
   (a) Supplier thinks her relative price has risen, so she produces more
   (b) With many producers thinking this way, Y will rise whenever P rises above P^e.



Each of the three models of agg. supply imply the relationship summarized by the *SRAS* curve & equation.

Suppose a positive *AD* shock moves output above its natural rate and *P* above the level people had expected.

Over time, P^e rises, *SRAS* shifts up, and output returns to its natural rate.



2.4 Pillips Curve

Proposition 2.4.0.1 (Inflation Dependent: inverse relation between inflation and unemployment)**.** *Phillips Curve (1960s)* states that

1. π depends on π^e
2. *cyclical unemployment* just is the deviation of the actual rate of unemployment from the natural rate: unemployment is related to unexpected movements in the inflation rate.
3. supply shocks, ν

From which we have

$$\pi = \pi^e - \beta\left(u - u^n\right) + \nu.$$

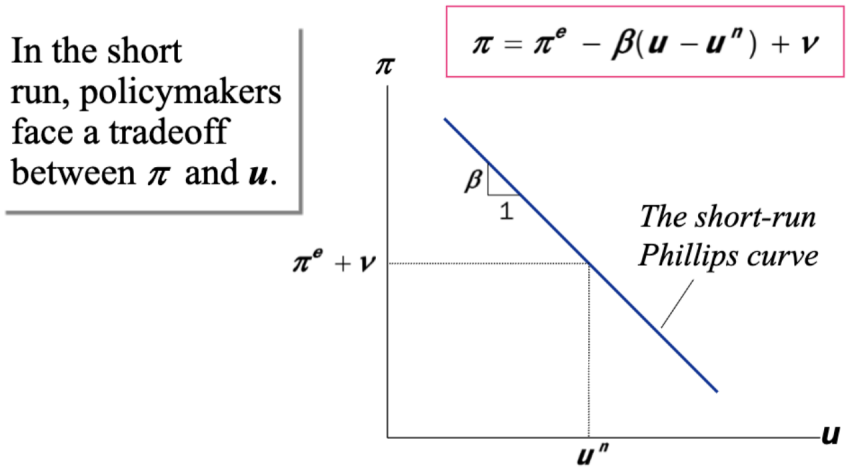
Model Derivation. Let ν be supply shocks. Then,

$$\begin{aligned} Y &= \overline{Y} + \alpha(P - P^e) \\ &\implies P = P^e + \left(\frac{1}{\alpha}\right)\left(Y - \overline{Y}\right) \\ &\implies P = P^e + \left(\frac{1}{\alpha}\right)\left(Y - \overline{Y}\right) + \nu \\ &\implies (P - P_{-1}) = (P^e - P_{-1}) + \left(\frac{1}{\alpha}\right)\left(Y - \overline{Y}\right) + \nu \end{aligned}$$

where P_{-1} is the price level of prior period; note that we have everything in ln

$$\implies \pi = \pi^e - \beta(u - u^n) + \nu \qquad \text{Okun's Law: } \left(\tfrac{1}{\alpha}\right)\left(Y - \overline{Y}\right) = -\beta(u - u^n)$$

□



2.5 Adaptive Expectations

Definition 2.5.1 (Adaptive Expectations)**.** An approach that assumes people form their expectations of future inflation based on recently observed inflation:

$$\pi^e = \pi_t = \pi_{t-1} \overset{\text{with weight}}{=} \sum w_i \pi_{-i}.$$

Then, we have

$$\pi = \pi_{-1} - \beta(u - u^n) + \nu.$$

Definition 2.5.2 (Inflation Inertia)**.** With $\pi = \pi_{-1} - \beta(u - u^n) + \nu$, inflation has inertia:

1. In the absence of supply shocks or cyclical unemployment, inflation will continue indefinitely at its current rate
2. Past inflation influences expectations of current inflation, which in turn influences the wages & prices that people set.

Proposition 2.5.2.1 (Two Causes of inflation fluctuation)**.**

1. *Cost-push inflation:*inflation resulting from supply shocks Adverse supply shocks typically raise production costs and induce firms to raise prices, "pushing" inflation up.
2. *Demand-pull inflation:* inflation resulting from demand shocks Positive shocks to aggregate demand cause unemployment to fall below its natural rate, which "pulls" the inflation rate up.

Definition 2.5.3 (Sacrifices Ratio)**.** The *sacrifice ratio* measures the percentage of a year's real GDP that must be foregone to reduce inflation by 1 percentage point:

$$\mathscr{S}_{ratio} := SR = \frac{\sum_{t=1}^{T} \left(\frac{Y_t - Y_t^*}{Y_t^*} \times 100 \right)}{\Delta\pi} \approx \frac{2 \cdot \sum_{t=1}^{T} (u_t - u^*)}{\Delta\pi} \text{ s.t. 2 is the Okun coefficient.}$$

2.6 Rational Expectations

Definition 2.6.1 (Rational Expectation)**.** People base their expectations on all available information, including information about current and prospective future policies.

Proposition 2.6.1.1 (Painless Disinflation)**.** It is argued by neoclassics that $\mathscr{S}_{ratio} \approx 0$. Suppose $u = u^n$ and $\pi = \pi^e = 6\%$. Suppose Fed announces a reduction of inflation from 6 to 2. If the announcement is credible, then π^e will fall, perhaps by the full 4 points. Then π can fall without an increase in u.

Claim 2.6.1.2 (Natural Rate Hypothesis)**.** Changes in aggregate demand affect output and employment only in the short run. In the long run, the economy returns to the levels of output, employment, and unemployment described by the classical model.

Claim 2.6.1.3 (Hysteresis)**.** An alternative hypothesis is *hysteresis,* the long-lasting influence of history on variables such as the natural rate of unemployment. As a result negative shocks may increase u^n so economy may not fully recover. There are two reasons to this long-lasting effect

1. The skills of cyclically unemployed workers may deteriorate while unemployed, and they may not find a job when the recession ends.

2. Cyclically unemployed workers may lose their influence on wage-setting; then, insiders (employed workers) may bargain for higher wages for themselves.

Results. The cyclically unemployed "outsiders" may become structurally unemployed when the recession ends; and so

$$\mathscr{S}_{ratio} = +\infty.$$

3 Dynamic Model

Remarks 3.0.0.0.1. In this course we study the simplified version of a Dynamic, Stochastic, General Equilibrum (DGSE) Model; built from

1. IS curve
2. Phillips Curve
3. Adaptive Expectations.

Our Model have three endogenous variables: output, inflation, real interest rate, nominal interest rate, and expected inflation.

3.1 Model Derivation

Definition 3.1.1 (Output)**.** The Demand for goods and services,

$$Y_t = \overline{Y}_t - \alpha\left(r_t - \rho\right) + \varepsilon_t,$$

where Y_t is the output, $\overline{Y}_t$ is the natural rate of output, ε_t is fiscal shock such that its average is zero, α is the measurement of the interest-rate of sensitivity of demand, r_t is the real interest rate and ρ is the natural rate of interest in absence of demand shocks $\alpha, \rho > 0$. Thus,

$$r_t = \rho \implies Y_t = \overline{Y}_t.$$

Definition 3.1.2 (Fisher Approximation)**.** The real interest rate is approximated,

$$r_t = i_t - \mathbb{E}_t\pi_{t+1},$$

where r_t is the *ex ante real rate*, i_t is the nominal interest rate, and $\mathbb{E}_t\pi_{t+1}$ is the expected inflation rate of $t+1$ formed in t.
The real interest rate is

$$1 + r_t = \frac{1+i}{1+\mathbb{E}_t\pi_{t+1}}.$$

Definition 3.1.3 (Philips Curve)**.** The Philips curve without Okun's Law,

$$\pi_t = \mathbb{E}_{t-1}\pi_t + \varphi\left(Y_t - \overline{Y}_t\right) + \nu_t.$$

Definition 3.1.4 (Adaptive Expectation)**.** Under adaptive assumption, we have

$$\mathbb{E}_t\pi_{t+1} = \pi_t.$$

Definition 3.1.5 (Monetary-Policy Rule: Taylor-Rule)**.** The nominal interest rate:

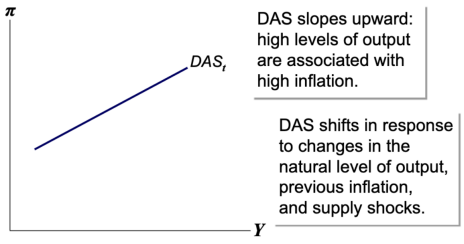
$$i_t = \pi_t + \rho + \theta_\pi\left(\pi_t - \pi_t^*\right) + \theta_Y\left(Y_t - \overline{Y}_t\right),$$

Where $\theta_{\pi,Y} > 0$ are the sensitivity to inflation and output with π_t^* being the inflation target. From which it follows that

$$\pi_t = \pi_t^* \text{ and } Y_t = \overline{Y}_t \implies i_t = \pi_t + \rho.$$

Definition 3.1.6 (The Dynamic Aggregate Supply Curve)**.** We have thus,

$$\pi_t = \mathbb{E}_{t-1}\pi_t + \varphi\left(Y_t - \overline{Y}_t\right) + \nu_t.$$



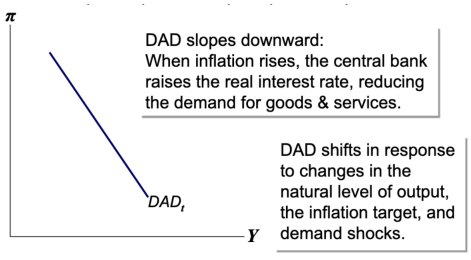
Definition 3.1.7 (The Dynamic Aggregate Demand Curve)**.** We have DAD,

$$Y_t = \overline{Yx}_t - A\left(\pi_t - \pi_t^*\right) + B\varepsilon_t, \text{ s.t. } A = \frac{\alpha\theta_\pi}{1+\alpha\theta_Y} > 0 \text{ and } B = \frac{1}{1+\alpha\theta_Y} > 0.$$

Model Derivation.

$$\begin{aligned}
Y_t &= \overline{Y}_t - \alpha\left(r_t - \rho\right) + \varepsilon_t && \text{Output} \\
&= \overline{Y}_t - \alpha\left(i_t - \mathbb{E}_t\pi_{t+1} - \rho\right) + \varepsilon_t && \text{Fisher Approx, } r_t = \mathbb{E}_t\pi_{t+1} \\
&= \overline{Y}_t - \alpha\left(i_t - \pi_t - \rho\right) + \varepsilon_t && \text{Adaptive Expectation, } \mathbb{E}_t\pi_{t+1} = \pi_t \\
&= \overline{Y}_t - \alpha\left(\pi_t + \rho + \theta_\pi\left(\pi_t - \pi_t^*\right) + \theta_Y\left(Y_t - \overline{Y}_t\right) - \pi_t - \rho\right) + \varepsilon_t \\
& && \text{Monetary Rule, } i_t = \pi_t + \rho + \theta_\pi\left(\pi_t - \pi_t^*\right) + \theta_Y\left(Y_t - \overline{Y}_t\right) \\
&= \overline{Y}_t - \theta_\pi\left(\pi_t - \pi_t^*\right) + \theta_Y\left(Y_t - \overline{Y}_t\right) + \varepsilon_t \\
&\overset{\text{solve for } Y}{\Longrightarrow} Y_t = \overline{Y}_t - \frac{\alpha\theta_\pi}{1+\alpha\theta_Y}\left(\pi_t - \pi_t\right. && \left. + \frac{1}{1+\alpha\theta_Y}\varepsilon_t \right. \\
&\implies Y_t = \overline{Y}_t - A(\pi_t - \pi_t^*) + B\varepsilon_t && \text{DAD; By constructed } A, B.
\end{aligned}$$

□



Remarks 3.1.7.0.1 (Model Parameters)**.** We have endogenous variables:

1. $Y_t :=$ output
2. $\pi_t :=$ inflation
3. $r_t :=$ real interest rate
4. $i_t :=$ nominal interest rate
5. $\mathbb{E}_t\pi_{t+1} :=$ Expected Inflation.

With exogenous variables:

1. $\overline{Y}_t :=$ natural level of output
2. $\pi_t^* :=$ central bank's target inflation rate
3. $\varepsilon_t =$ Demand Shock
4. $\nu_t =$ Supply Shock

And predetermined variable,

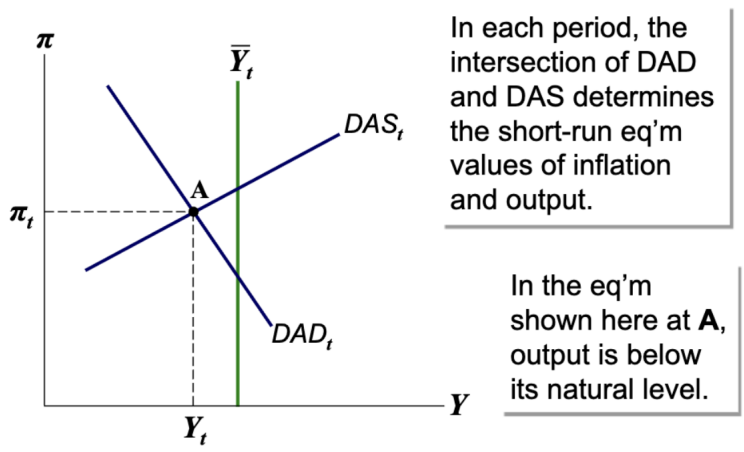
$$\pi_{t-1} := \text{prior inflation}$$

Proposition 3.1.7.1 (Long-run equilibrium)**.** The model arrives at equilibrium if

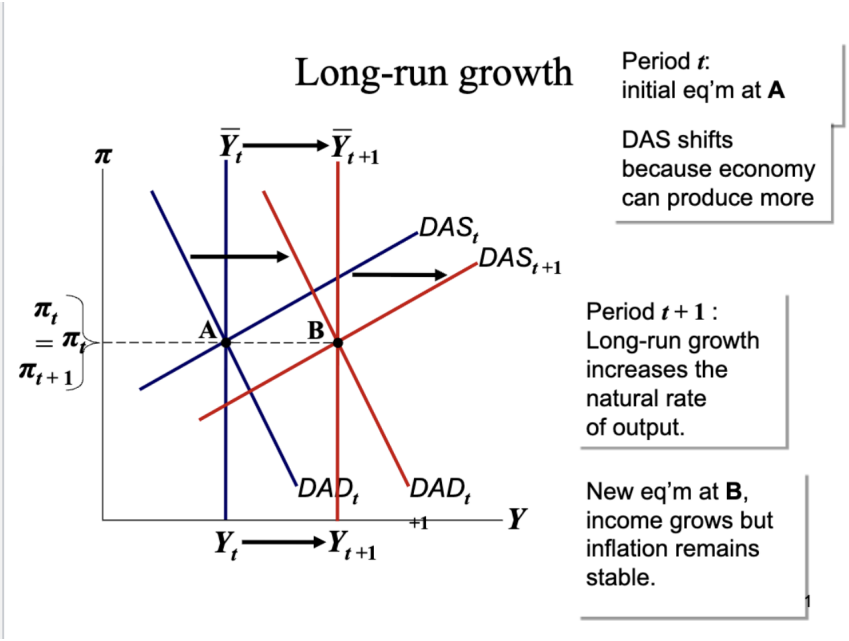
$$\varepsilon_t = \nu_t = 0, \pi_t = \pi_t^*, r_t = \rho_t, \mathbb{E}_t\pi_{t+1} = \pi_t^*, \text{ and } Y_t = \overline{Y}_t,$$

i.e., when there are no shocks and inflation is constant.

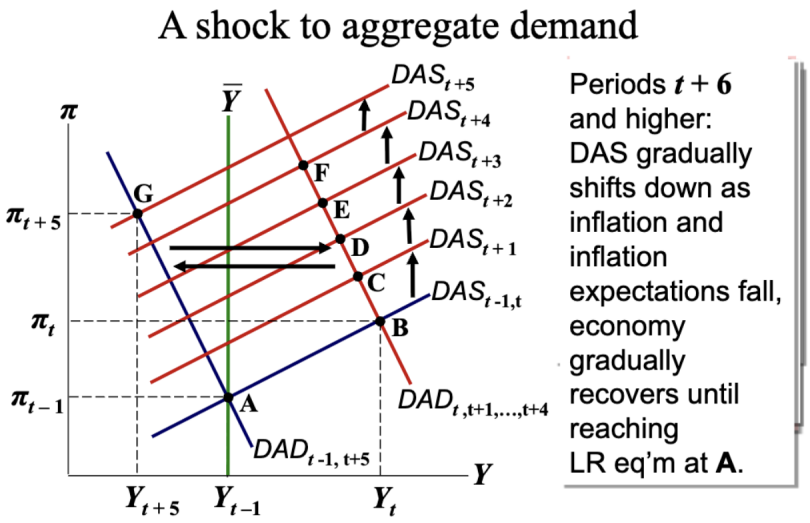
Proposition 3.1.7.2 (Short Run Equilibrium)**.** The model is in SR equilibrium at the intersection of DAD and DAS.



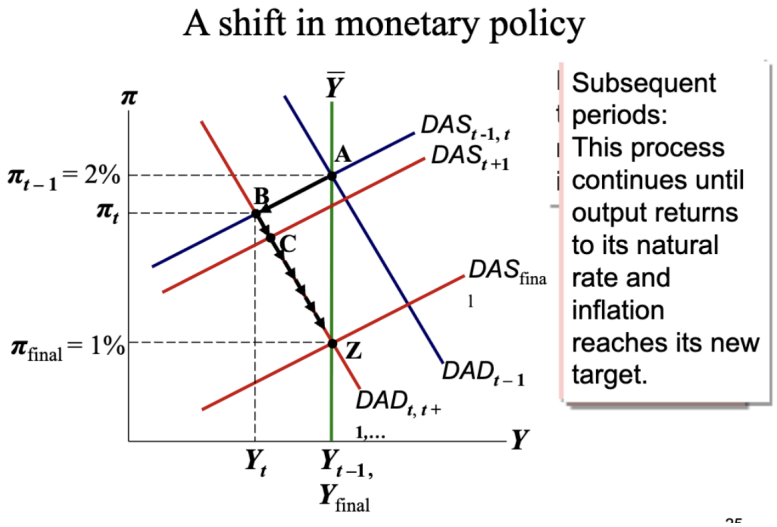
Proposition 3.1.7.3 (Long Run Growth)**.** In LR, growth without inflation



Proposition 3.1.7.4 (A shock to aggregate demand)**.** Suppose a shock occurs to DAD; then, in t the equilibrium is at B, and subsequently,

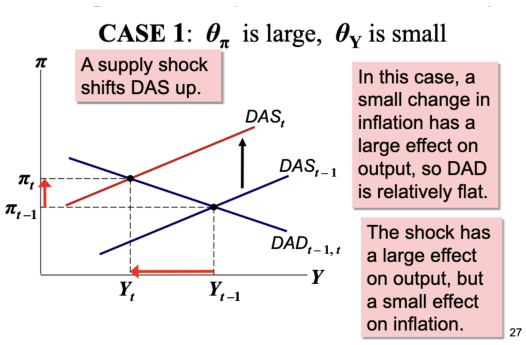


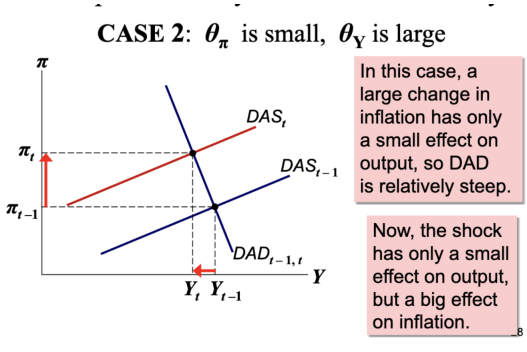
Proposition 3.1.7.5 (Monetary Shock)**.** Suppose a monetary shock occurse such that we move from A to B. Then,



3.2 Application: θ_π v.s. θ_Y

Remarks 3.2.0.0.1 (Variability)**.** A supply shock reduces output (bad) and raises inflation (also bad). Thus, the central bank faces a tradeoff between these "bads" – it can reduce the effect on output, but only by tolerating an increase in the effect on inflation....
Consider two cases:





4 Stabilization

Remarks 4.0.0.0.1. This section aims to answer two questions:

1. Should policy be active or passive?
2. Should policy be by rule or discretion

4.1 Active v.s. Passive

Proposition 4.1.0.1 (Active Policy)**.** Keynesian (1928/1932) and New Keynesian economists advocate for active policy. Recessions cause economic hardship for millions of people. The Employment Act of 1946: "It is the continuing policy and responsibility of the Federal Government to...promote full employment and production."

Proposition 4.1.0.2 (Arguments Against Active Policy)**.**

1. inside lag: the time between the shock and the policy response.
   (a) takes time to recognize the shock
   (b) take time to implement policy, especially fiscal policy
2. outside lag: the time it takes for policy to affect the economy

Definition 4.1.1 (Automatic Stabilizers)**.** Policies that stimulate or depress the economy when necessary, without any deliberate policy. These policies are designed to reduce the lags associated with stabilization policy. For example,

1. income tax
2. unemployment insurance
3. welfare

Proposition 4.1.1.1 (Forecasting the Macroeconomy)**.** There are two ways economists generate forecasts:

1. *Leading economic indicators:* data series that fluctuate in advance of the economy
2. *Macroeconometric models* Large-scale models with estimated parameters that can be used to forecast the response of endogenous variables to shocks and policies

Proposition 4.1.1.2 (The Lucas critique)**.** The Lucas critique is attributed to Robert Lucus who won Nobel Prize in 1995 for rational expectations. Forecasting the effects of policy changes has often been done using models estimated with historical data. Lucas pointed out that such predictions would not be valid if the policy change alters expectations in a way that changes the fundamental relationships between variables. For example, prediction (based on past experience): An increase in the money growth rate will reduce unemployment. The Lucas critique points out that increasing the money growth rate may raise expected inflation, in which case unemployment would not necessarily fall.

Remarks 4.1.1.2.1. Most economists agree, though, that the U.S. economy (and Canadian Economy) has become much more stable since the late 1980.

4.2 Rule v.s. Discretion

Definition 4.2.1 (Policy conducted by rule)**.** Policymakers announce in advance how policy will respond in various situations, and commit themselves to following through.

Definition 4.2.2 (Policy conducted by discretion)**.**
As events occur and circumstances change, policymakers use their judgment and apply whatever policies seem appropriate at the time.

Proposition 4.2.2.1 (Arguments for Rules)**.**

1. *Distrust of policymakers and the political process,* e.g., misinformed politicians, and politicians' interests sometimes not the same as the interests of society
2. *The time inconsistency of discretionary policy:* A scenario in which policymakers have an incentive to renege on a previously announced policy once others have acted on that announcement. As a consequence Destroys policymakers' credibility, thereby reducing effectiveness of their policies.
   Examples.
   (a) To encourage investment, govt announces it will not tax income from capital. But once the factories are built, govt reneges in order to raise more tax revenue.
   (b) To reduce expected inflation, the central bank announces it will tighten monetary policy. But faced with high unemployment, the central bank may be tempted to cut interest rates.

Definition 4.2.3 (Monetary Policy Rules)**.**

1. Costant Money Supply Growth Rate
2. Target Growth Rate of Nominal GDP
3. Target The Inflation Rate
4. The Taylor Rule: target the bank rate based on
   (a) inflation rate
   (b) gap between actual and full employment GDP

$$i_t = r_t + \rho + \theta_\pi \left(\pi_t - \pi_t^*\right) - \theta_y(\text{GDP Gap})$$

Definition 4.2.4 (Central Bank Independence)**.**

1. A policy rule announced by central bank will work only if the announcement is credible.
2. Credibility depends in part on degree of independence of central bank.

Proposition 4.2.4.1 (Time inconsistency and the tradeoff between Inflation and Unemployment)**.** The Philips curve gives

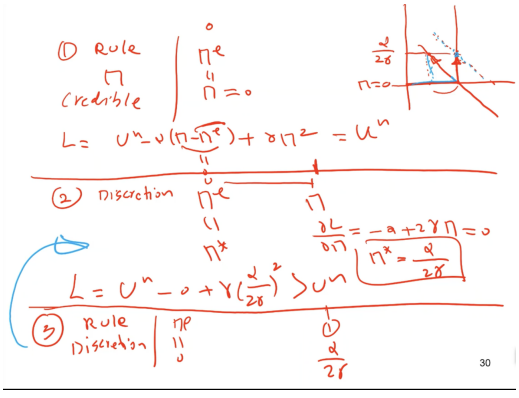
$$\begin{aligned} \pi &= \pi^e - \beta\left(u - u^n\right) + \nu \\ \implies u &= u^n - \alpha\left(\pi - \pi^e\right) \qquad \text{suppose 0 supply shock} \end{aligned}$$

Suppose the Bank *loss function* (low inflation and low unemployment) is characterized by:

$$L\left(u, \pi\right) = u + \gamma\pi^2$$

where γ represents how much the Bank dislikes inflation relative to unemployment. Thus, we have from Philips Curve,

$$\begin{aligned} L\left(u, \pi\right) &= \left[u^n - \alpha\left(\pi - \pi^e\right)\right] + \gamma\pi^2 \\ &\overset{\min \text{L}}{\implies} \frac{\partial L}{\partial \pi} = 0 \iff \pi = \frac{\alpha}{2\gamma}. \end{aligned}$$



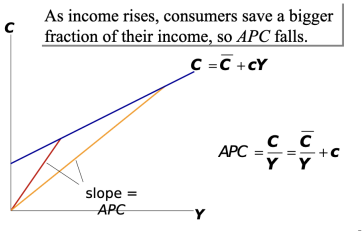
5 Micro-Basis: Consumption

5.1 Keynes: Consumption and Current Income

Definition 5.1.1 (Keynes's Conjecture)**.** Keynes assumes that

1. $MPC \in (0, 1)$.
2. $Y \uparrow \implies APC = \frac{C}{Y} \downarrow$.
3. Income is the main determinant of consumption.

Definition 5.1.2 (Keynesian Consumption Function)**.** The Keynesian consumption function is given:



Remarks 5.1.2.0.1 (Early Success)**.** Early empirical studies support Keynes's Conjecture in that higher income household

1. Consume less $\implies MPC > 0$
2. Save more $\implies MPC < 1$
3. As $Y \uparrow APC \downarrow$.

A very strong correlation is established between income and consumption, i.e., income seemed to be the main determinant of consumption

Proposition 5.1.2.1 (Kuznets Puzzle)**.** Simon Kuznets showed that $\frac{C}{Y}$ was very stable across 100 years (1835 - 1935) of time series data, which is inconsistent with the claim that APC will fall insofar as Y grows. In fact, C grows at the same rate as Y.

Proposition 5.1.2.2 (SR v.s. LR consumption)**.** According to Kuznets puzzle, therefore, APC declines in SR and is constant in the LR (shown by Fisher). So both Keynes and Kuznets were right in different time frame

5.2 Fisher: Intertemporal Choice

Remarks 5.2.0.0.1. Fisher assumes that consumer is forward-looking and chooses consumption for the present and future to maximize lifetime satisfaction. Consumer's choices are subject to an *intertemporal budget constraint,* a measure of the total resources available for present and future consumption.

Definition 5.2.1 (The basic two-period model)**.** Suppose we divide time into two periods: period 1- the present and period 2 - the future. Then, period 2 budget constraint is

$$C_2 = Y_2 + (1+r)S,$$

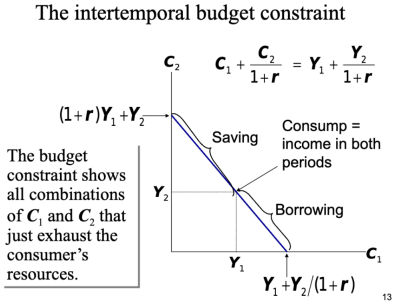
i.e., all of the remaining income and $S = Y_1 - C_1$ and so

$$C_2 = Y_2 + (1+r)(Y_1 - C_1) \iff (1+r)C_1 + C_2 = Y_2 + (1+r)Y_1.$$

I.e., divide by $1+r$ yeilds

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r},$$

where the present value of life time consumption is the present value of life time income.



Note that

$$\begin{aligned} C_1 + \frac{C_2}{1+r} &= Y_1 + \frac{Y_2}{1+r} \\ &\iff C_2 = (1+r)(Y_1 - C_1) + Y_2 \\ &\iff \frac{\partial C_2}{\partial C_1} = -(1+r) \qquad \text{slope of budget constraint} \end{aligned}$$

Proposition 5.2.1.1 (Change of r)**.** Suppose an increase in r occurs. Then,

	SE	IE borrower	IE savor	IE aggregate
C_1	$(-)$	$(-)$	$(+)$	?
C_2	$(+)$	$(-)$	$(+)$	$(+)$

Definition 5.2.2 (Binding)**.** The borrowing constraint is not binding if the consumer's optimal C_1 is less than Y_1. Else, binding: but since the consumer cannot borrow, the best he can do to settle at a corner solution.

Proposition 5.2.2.1 (Argument Summery: Fisher v. Keynes)**.**

1. Keynes: Current consumption depends only on current income.
2. Fisher: Current consumption depends only on the present value of lifetime income. The timing of income is irrelevant because the consumer can borrow or lend between periods.

5.3 Modigliani: Life-Cycle Hypothesis

Remarks 5.3.0.0.1. The life-cycle hypothesis is due to Franco Modigliani (1950s). Fisher's model says that consumption depends on lifetime income, and people try to achieve smooth consumption. The LCH says that income varies systematically over the phases of the consumer's "life cycle," and saving allows the consumer to achieve smooth consumption.

Definition 5.3.1 (Life-Cycle Model)**.** We assume a zero real interest rate and that consumption-smoothing is optimal. Let W be initial wealth, Y be annual income until retirement (assumed constant), R be the number of years until retirement, and T be the life time in years. And so life time resources is defined as

$$\text{lifetime resources} = W + RY.$$

To achieve consumption smoothing, she divides resources equally over time

$$C = \frac{W + RY}{T} := \alpha W + \beta Y,$$

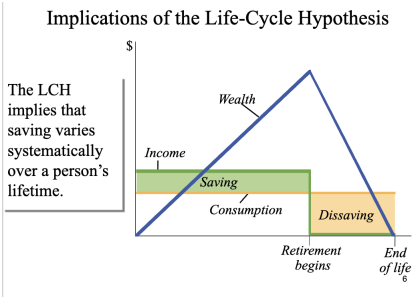
where $\alpha = \frac{1}{T}$ is the *marginal propensity to consume out of wealth* and $\beta = \frac{R}{T}$ is the *marginal propensity to consume out of income*. Note that $\beta > \alpha$.

Proposition 5.3.1.1 (LCH solves the consumption puzzle)**.** The life-cycle consumption function implies that

$$APC = C/Y = \alpha \left(\frac{W}{Y}\right) + \beta$$

1. Across households, income varies more than wealth, so high-income households should have a lower APC than low-income households.

2. Over time, aggregate wealth and income grow together, causing APC to remain stable.



5.4 Friedman: Permanent Income Hypothesis

Remarks 5.4.0.0.1. The Permenant Income Hypothesis is due to Milton Friedman (1957). He posits that

$$Y = Y^P + Y^T,$$

where Y is *current income* and Y^P is *permanent income*, i.e., average income, which people expect to persist into the future and Y^T is *transitory income*, i.e., temporary deviations from average income

Proposition 5.4.0.1 (PIH solves the consumption puzzle)**.** Consumers use saving and borrowing to smooth consumption in response to transitory changes in income. The PIH consumption function is given:

$$C = \alpha Y^P,$$

where α is the fraction of permanent income that people consume per year. From which it follows that

$$APC = \frac{C}{Y} = \alpha \frac{Y^P}{Y},$$

Note that in SR, if high-income households have higher transitory income than low-income households, APC is lower in high-income households. Whereas in the long run income variation is due mainly (if not solely) to variation in permanent income, which implies a stable APC.

Definition 5.4.1 (LCH v.s. PIH)**.** Both: people try to smooth their consumption in the face of changing current income.

1. LCH: current income changes systematically as people move through their life cycle.
2. PIH: current income is subject to random, transitory fluctuations

5.5 Hall: Random Walk Hypothesis

Remarks 5.5.0.0.1. RWH is due to Robert Hall (1978). Based on Fisher's model and PIH, in which forward-looking consumers base consumption on expected future income, Hall adds the assumption of *rational expectations*, that people use all available information to forecast future variables like income.

Definition 5.5.1 (The Random-Walk Hypothesis)**.** If PIH is correct and consumers have rational expectations, then consumption should follow a *random walk*: changes in consumption should be unpredictable.

1. A change in income or wealth that was anticipated has already been factored into expected permanent income, so it will not change consumption.

2. Only unanticipated changes in income or wealth that alter expected permanent income will change consumption.

Thus, *if consumers obey the PIH and have rational expectations, then policy changes will affect consumption only if they are unanticipated.*

6 Deficits and Debt

6.1 Government Debt

Proposition 6.1.0.1 (Government Debt is Different)**.**

1. *Incorrect analogy between household and government debt:* "Government debt represents an obligation that will have to be repaid in the future."
2. *Correct view:* the government is in principle immortal and never has to repay the debt. It repays maturing bonds by selling new bonds (i.e., they are required to pay the interest)
   (a) If future generations won't buy the debt, the Central Bank will.
   (b) If foreigners won't buy government debt $ depreciates, but government still doesn't have to pay

Remarks 6.1.0.1.1 (Is Government Debt Free)**.** Government Debt is not free:

1. Tax rates are higher in order to pay the interest on the debt
2. Debt and Deficits lower national saving which leads to higher real interest rates and potentially bad trade deficits.
3. High Debt may lead to an increase in inflationary bias.

Proposition 6.1.0.2 (Debt Leads to Higher Tax Rates)**.** While the principle never has to be repaid, the current interest does:

1. the payment of current interest is a transfer of purchasing power from taxpayers to bondholders.
2. Taxes affect prices and distort the allocation of resources

Proposition 6.1.0.3 (Debt and Deficit Lower Saving)**.**

1. Large Open economy: Fiscal expansion leads to an increase in the real exchange rate and trade deficit.
2. Mankiw: In the long run, the smaller national saving ... would mean a smaller capital stock and a greater foreign debt. Therefore, the output of the nation would be smaller and a greater share owed to foreigners.

Proposition 6.1.0.4 (High Debt may lead to an increase in inflationary bias)**.** We know that inflationary bias depends on the central bank's aversion to inflation. Debtors gain with an unexpected increase in inflation. The existence of debt could reduce a politically dependent Central Bank's aversion to inflation and raise inflationary bias.

Definition 6.1.1 (Debt to GDP- Relative Debt)**.** The costs to the government debt depend on *the size of the debt relative to GDP, i.e.,*

$$\frac{Debt}{GDP}$$

It is correct to be concerned about the government debt, but focus on the relative debt

Remarks 6.1.1.0.1 (Contemporary Problems)**.** However...

1. The population is aging.
2. Health care costs are rising.
3. Spending on entitlements like Social Security is growing.
4. US: Deficits and the debt are projected to significantly increase...

6.2 Measurement of Budget Deficit

Remarks 6.2.0.0.1 (Problems Measuring the Deficit)**.** We encounter problems when measuring:

1. Inflation
2. Capital Assets
3. Uncounted Liabilities
4. The Business Cycle

Definition 6.2.1 (Government Defiicit)**.** The government deficit, D is given by

$$D := [G + iB] - T = \Delta B,$$

where G is government spending, B is government debt, T is tax and i is the nominal interest on government debt.

Proposition 6.2.1.1 (Counter - Inflation)**.** Let D^n be nominal deficit with r being the real interest on government debt with D^r being the real deficit, then

$$\begin{aligned} D^n &= [G + iB] - T = \Delta B \\ &\implies D^r = D^n - \pi B \qquad \text{by fisher approx: } i = r + \pi. \end{aligned}$$

From which it follows that whenever the real deficit is zero, we must have

$$\Delta B = \pi B \iff \frac{\Delta B}{B} = \pi, \text{ i.e., debt grows at the rate of inflation.}$$

Theorem 6.2.1.2 (Constant Debt-to-GDP)**.** The Debt-to-GDP ratio is 0 if

$$\frac{\Delta \text{GDP}_{real}}{\text{GDP}_{real}} = 0.$$

Proof. Let DTG be defined

$$DTG = \frac{B}{GDP}.$$

Then,

$$\begin{aligned} ln(DTG) &= \ln(B) - \ln(GDP) \\ &\implies \frac{\Delta DTG}{DTG} = \frac{\Delta B}{B} - \frac{\Delta GDP}{GDP} = 0 \iff \text{GDP growth} = \pi. \end{aligned}$$

□

Proposition 6.2.1.3 (Counter - Capital Assets)**.** We currently define deficit as the change in debt:

$$D = \Delta B.$$

It can be argued that the assets should be taken into account, i.e., *capital budgeting*:

$$D = \Delta B - \Delta \text{Assets}.$$

Example. Suppose govt sells an office building and uses the proceeds to pay down the debt.

1. under current system, deficit would fall
2. under capital budgeting, deficit unchanged, because fall in debt is offset by a fall in assets.

The problem then becomes **determining which govt expenditures count as capital expenditures.**

Proposition 6.2.1.4 (Counter- Uncounted Liabilities)**.** Current measure of deficit omits important liabilities of the government:

1. future pension payments owed to current govt workers.
2. future Social Security payments
3. contingent liabilities, e.g., covering federally insured deposits when banks fail (Hard to attach a dollar value to contingent liabilities, due to inherent uncertainty.)

Proposition 6.2.1.5 (Counter - The Business Cycle)**.** The deficit varies over the business cycle due to automatic stabilizers (unemployment insurance, the income tax system). These are not measurement errors, but do make it harder to judge fiscal policy stance. E.g., is an observed increase in deficit due to a downturn or an expansionary shift in fiscal policy? **Solution**: *cyclically adjusted budget deficit* (aka "full-employment deficit") – based on estimates of what govt spending and revenues would be if economy were at the natural rates of output and unemployment.

6.3 Government Debt: Keynesian View

Remarks 6.3.0.0.1. Is the government debt really a problem?

Proposition 6.3.0.1 (Theoretical Framework-Long Run: Closed Economy)**.** Suppose there is a tax cut with corresponding increase in government debt. Then, in SR:

$$\uparrow Y, \downarrow u.$$

In LR:

1. Y and u back at their natural rates
2. closed economy: $\uparrow r, \downarrow I$
3. open economy: $\uparrow \varepsilon, \downarrow NX$.

Very long run: slower growth until economy reaches new steady state with lower income per capita.

Theorem 6.3.0.2 (Fiscal expansion increases the real interest rate Long Run: Closed Economy)**.** We know

$$Y - C - G - I = 0 \iff NS - I = 0.$$

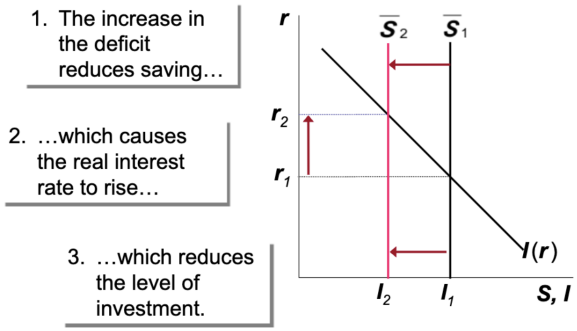
From which it follows that $NS = I$. In particular,

$$\begin{aligned} NS &= Y - C - G \\ &= Y - C - G - T + T - G \\ &= S_{private} + D = S_{private} + S_{public} \qquad \text{where } D := T - G \end{aligned}$$

Note that

1. $Y = F(K, L)$. The output Y of the economy is fixed by the
2. $C = f(Y - T)$.Consumption is positively related to disposable
3. $I = I(r)$. vestment is negatively related to the interest rate. income (Y-T). factors of production and the production function.

Thus,
$\implies$ Budgetary fiscal policy determines saving. - High government expenditures and low tax rates result in sizeable deficits and lower national saving.
$\implies$ investment depends on the interest rate r.
$\implies$ High interest rates decrease the number of profitable projects.
Suppose an fiscal expansion is implemented. Then,



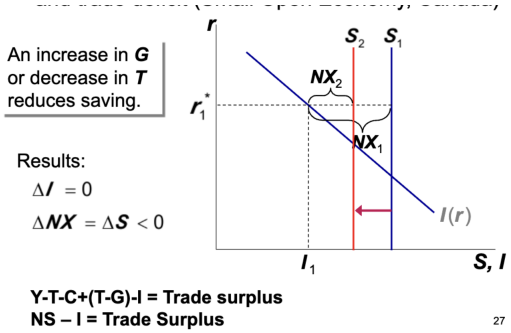
Theorem 6.3.0.3 (Open Economy)**.** We have

$$Y - C - G - I = NX \implies NS - I = NX.$$

In other words,

$$NX = [Y - c(Y - T) - G] - I(r) \implies NX = NS - I(r).$$

Suppose there is an increase in G or decrease in T reduces saving.



Example. Consider an economy described by the following

$$\begin{gathered} Y = C + I + G + NX, \quad \bar{Y} = 1500 \\ G = 300, \qquad\qquad T = 300 \\ C = 25 + 0.25Y^d \\ I = 1000 - 50r \\ NX = 250 - 10\varepsilon \\ NFI = 600 - 50r \end{gathered}$$

Find real interest rate, national saving, real exchange rate in the equilibrium.

Solution. First we note that

$$S^N - I = NX = NFI.$$

From which it follows that

$$\begin{aligned} S^N - I = Y - C - G &= 1500 - (25 + 0.25\,(1500 - 300)) - 300 - 1000 + 50r = NFI = 600 - 50r \\ &\implies -125 + 50r = 600 - 50r \implies r = 7.25 \text{ and } S^N = 875 \\ &\implies NFI = 237.5 = NX \implies \varepsilon = 1.75 \\ &\implies I = 637.5 \qquad \text{as needed.} \end{aligned}$$

Consider $T_1 = 200$. Then,

$$\begin{aligned} S^N - I = 1500 - 350 - 300 - 1000 + 50r = NFI = 600 - 50r \\ \implies r = 7.5 \text{ and } S^N = 850 \\ \implies NFI = 225 = NX \implies \varepsilon = 2.5 \\ \implies I = 625 \end{aligned}$$

as needed.

■

6.4 Government Debt: Ricardian View

Proposition 6.4.0.1 (Ricardian View)**.** Consumers are forward-looking, know that a debt-financed tax cut today implies an increase in future taxes that is equal—in present value—to the tax cut. Thus, the tax cut does not make consumers better off, so they do not raise consumption. - Save the full tax cut in order to repay the future tax liability. **Result:** Private saving rises by the amount public saving falls, leaving national saving unchanged.

Proposition 6.4.0.2 (Fisher Model on Debt)**.** Let T_1 be tax in period $1, T_2$ period 2; similarly G_1 and G_2 denote government expenditure in respective periods. Let

$$D = G_1 - T_1$$

be government deficit in period 1. Note that $D > 0$ iff we have a deficit and else surplus. Then, we have period 2 budget constraint

$$T_2 = (1+r)D + G_2 \implies T_2 - G_2 = (1+r)D.$$

Rearrangement gives

$$(1+r)T_1 + T_2 = (1+r)G_1 + G_2 \iff T_1 + \frac{T_2}{1+r} = G_1 + \frac{G_2}{1+r}.$$

Thus, $\downarrow T_1$ by $\Delta T \implies \uparrow T_2 = \Delta T(1+r)$. On the other hand for household budget, in period 1 saving,

$$S = (Y_1 - T_1) - C_1.$$

Period 2 budget constraint:

$$C_2 = (Y_2 - T_2) + (1+r)\,S = (Y_2 - T_2) + (1+r)\,(Y_1 - T_1 - C_1)\,.$$

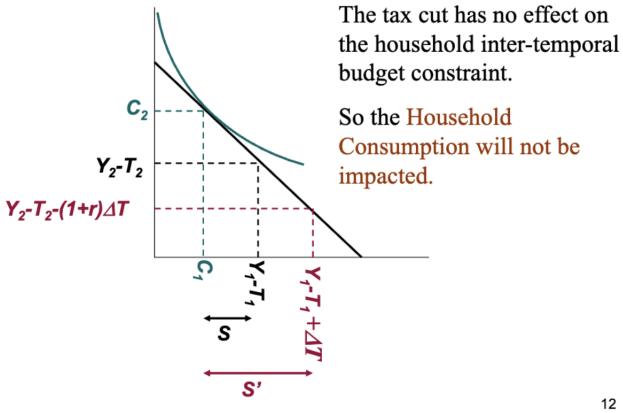
From which it follows that

$$(1+r)\,C_1 + C_2 = (1+r)\,(Y_1 - T_1) + (Y_2 - T_2) \iff C_1 + \frac{C_2}{1+r} = Y_1 - T_1 + \frac{Y_2 - T_2}{1+r}.$$

Proposition 6.4.0.3 (Change in Taxes)**.** If government cuts T_1 by amount ΔT, then it must raise T_2 by $(1+r)\Delta T$:

$$\begin{aligned} C_1 + \frac{C_2}{1+r} &= (Y_1 - T_1 + \Delta T) + \frac{Y_2 - T_2 - (1+r)\,\Delta T}{1+r} \\ &= (Y_1 - T_1) + \Delta T - \Delta T + \frac{Y_2 - T_2}{1+r}. \end{aligned}$$

The change in taxes cancels out, and so the consumer's intertemporal budget constraint is *unchanged*.



Thus, according to Ricardian view, tax cut just is a rescheduling of taxes into the future (change in endowment), and so consumption remain unchanged.

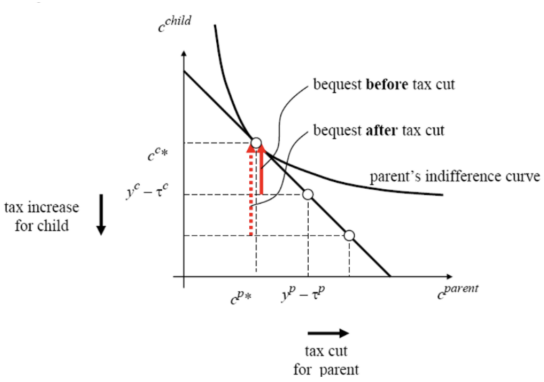
Proposition 6.4.0.4 (Summury of View)**.**

Effects of $1 Tax Cut

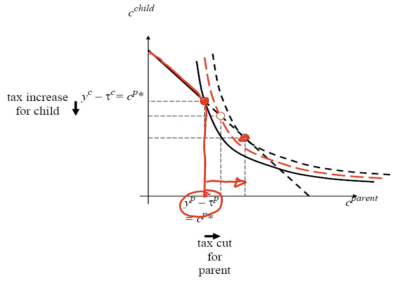
	S_{gov} = T-G	S_{Priv} = Y-T-C	S = Y-C-G = I + NX
Traditonal	down $1	up by $(1-mpc)	down by $mpc
Ricardian	down $1	up $1	0

Remarks 6.4.0.4.1 (Ricardian Limits)**.**

1. *Myopia:* Not all consumers think that far ahead, so they see the tax cut as a windfall
2. *Borrowing Constraint:* Some consumers are not able to borrow enough to achieve their optimal consumption, and would therefore spend a tax cut.
3. *Furture Generations:* Baro (1944) argues that if consumers expect that the burden of repaying a tax cut will fall on future generations, then a tax cut now makes them feel better off, so they increase spending.



Existence of bequests is not evidence for parents caring about their children's consumption in a similar way that they care about their own future consumption. Why? because most bequests turn out to be "accidental" (not voluntary). How do we know this? One way to see is that leaving bequests is a very expensive way of leaving resources for children (highly taxed). Parents could just transfer those resources, if they wanted to, to their children while they are alive. However, for Barro's reasoning to work, it must be optimal for parents to leave positive bequests. Because of economic growth, children's incomes are likely to be higher than parents' incomes, and the optimal thing is to try to get resources from kids or be at a corner. In this case, a tax cut will not be passed on to kids.



Proposition 6.4.0.5 (Evidence Against Ricardian Equivalence)**.**

1. Early 1980s: Huge Reagan tax cuts caused deficit to rise. National saving fell, the real interest rate rose, the exchange rate appreciated, and NX fell.
2. 1992: President George H.W. Bush reduced income tax withholding to stimulate economy. This merely delayed taxes but didn't make consumers better off. Yet, almost half of consumers used part of this extra take-home pay for consumption.
3. Proponents of R.E. argue that the Reagan tax cuts did not provide a fair test of R.E.
   (a) Consumers may have expected the debt to be repaid with future spending cuts instead of future tax hikes
   (b) Private saving may have fallen for reasons other than the tax cut, such as optimism about the economy
4. Because the data is subject to different interpretations, both views of govt debt survive

7 Micro-Basis: Investment

7.1 Business Fixed Investment

Definition 7.1.1 (Business Fixed Investment)**.** Businesses' spending on equipment and structures for use in production

Definition 7.1.2 (Neoclassical Model of Investment)**.** The standard model of business fixed investment: investment depends on:

1. $MP_K = \frac{\partial y}{\partial k}$ s.t. $y = f(K, L)$
2. interest rate
3. tax rules affecting firms

In which,

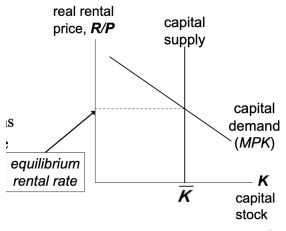
$$I = \Delta K + \delta K = I_n \left(MP_k - \frac{P_k}{P} (r + \delta) \right) + \delta K.$$

Definition 7.1.3 (Two Types of firm)**.** Suppose there are two types of firms,

1. *Production firms* who rent the capital they use to produce goods and services.
2. *Rental firms* whose capital is rented by production firms.

Definition 7.1.4 (The Capital Rental Market)**.** In the capital rental market, production firms must decide how much capital to rent; to be competitive, they rent capital such that

$$MP_K = \frac{\partial y}{\partial k} = \frac{R}{P}.$$



In particular, for *Cobb-Douglas* production function,

$$Y = F(K, L) = AK^{\alpha} L^{1-\alpha} \implies MP_K = \frac{R}{P} = \alpha A (\frac{L}{K})^{1-\alpha}.$$

From which it follows that $\downarrow K, \uparrow L, \uparrow A \implies \uparrow \frac{R}{P}$. Thus, we can claim that

$$I = f(K^-, A^+, L^+).$$

Definition 7.1.5 (Rental firms' investment decisions: the cost of capital)**.** Rental firms invest in new capital when the benefit, $\frac{R}{P}$ of doing so exceeds the cost. Note that here $\frac{R}{P}$ becomes the income from which the rental firms earn. The components of the cost of capital can be summarized with P_k being the price of the capital:

1. *interest cost*: $i \times P_k$ where $P_k :=$ nominal price of capital
2. *depreciation cost*: $\delta \times P_K$, where $\delta =$rate of depreciation
3. *capital loss*: $-\Delta P_k$, note that a capital gain, $\Delta P_k > 0$ reduces cost of K.

Thus,

$$\text{Nominal Cost of Capital} = iP_k + \delta P_k - \Delta P_k = P_k \left(i + \delta - \frac{\Delta P_k}{P_k} \right).$$

Example. Car Rental Company (with cars being the capital) Suppose $P_K = \$10{,}000$, $i = 0.10$, $\delta = 0.20$, and $\frac{\Delta P_K}{P_K} = 0.06$

Then,

interest cost	= \$1000
depreciation cost	= \$2000
capital loss	= –\$600
total cost	= \$2400

Now, suppose

$$\frac{\Delta P_k}{P_k} = \pi.$$

Then, the nominal cost of capital equals, by fisher's approximation

$$P_k(i + \delta - \pi) = P_k(r + \delta).$$

Thus, in real term (real cost of capital),

$$\frac{P_k}{P}(r + \delta).$$

And thus,

Proposition 7.1.5.1 (Real Cost of Capital Dependence)**.** Real Cost of Capital depends on $\frac{P_k}{P}, r, \delta$. where $\frac{P_k}{P}$ is the relative price.

Proposition 7.1.5.2 (The rental Firm's Profit Rate)**.** A firm's net investment depends on its profit rate,

$$\text{profit rate} = \frac{R}{P} - \frac{P_k}{P}(r + \delta) = MP_k - \frac{P_k}{P}(r + \delta).$$

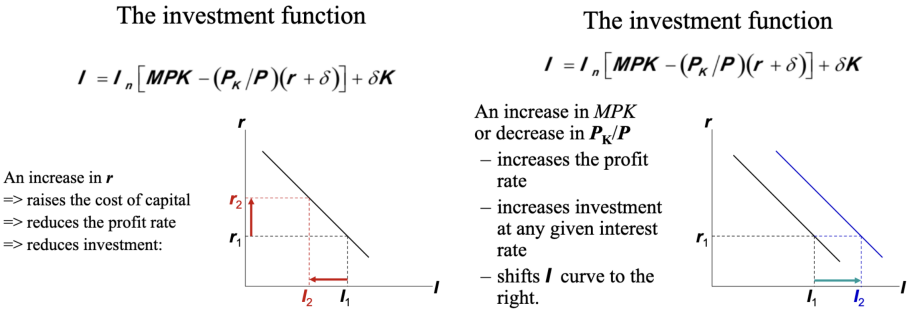
It follows that if the profit rate > 0, then increasing K is profitable; else, reducing the capital stock (i.e., ceasing replacement as it depreciates).

Proposition 7.1.5.3 (Net, Gross Investment)**.** Hence,

$$\text{Net investment} = \Delta K = I_n\left(\text{profit rate}\right) = I_n\left(MP_k - \frac{P_k}{P}\left(r+\delta\right)\right)$$

where I_n is a function that describes how net investment responds to the incentive to invest. On the other hand, total spending includes the replacement of depreciated K :

$$\text{Gross Investment} = \text{Net Investment} + \delta K = \Delta K + \delta K = I_n\left(MP_k - \frac{P_k}{P}\left(r+\delta\right)\right) + \delta K.$$



7.2 Taxes and Investment

Definition 7.2.1 (Types of Taxes)**.** The two most important taxes affecting investment are

1. Corporate income tax: $\Pi = \left(MP_K - \frac{R}{P}\right) - T \times \left(MP_K - \frac{R}{P}\right) = \left(MP_K - \frac{R}{P}\right) \times (1-T)$.
2. Investment tax credit : $P_K^* = P_K \times (1-T)$.

Proposition 7.2.1.1 (Impact on Investment: Corporate Income Tax)**.** Impact on investment depends on definition of "profit".

1. In our definition (rental price minus cost of capital), depreciation cost is measured using current price of capital, and the CIT would not affect investment
2. But, the legal definition uses the historical price of capital.
3. If P_K rises over time, then the legal definition understates the true cost and overstates profit, so firms could be taxed even if their true economic profit is zero. Thus, corporate income tax discourages investment.

Proposition 7.2.1.2 (Impact on Investment: Investment Tax Credit)**.** The ITC reduces a firm's taxes by a certain amount for each dollar it spends on capital. Hence, the ITC effectively reduces P_K which increases the profit rate and the incentive to invest.

Definition 7.2.2 (Tobin's q)**.** Let Tobin's q be defined

$$q := \frac{\text{Market Value of Installed Capital}}{\text{Replacement Cost of Installed Capital}} = \frac{MP_K}{\frac{R}{P}}.$$

Where the numerator is the stock market value of the economy's capital stock. The denominator is the actual cost to replace the capital goods that were purchased when the stock was issued. If $q > 1$, firms buy more capital to raise the market value of their firms; if $q < 1$, firms do not replace capital as it wears out. The stock market value of capital depends on the current and expected future profits of capital. If $MPK >$ cost of capital, then profit rate is high, which drives up the stock market value of the firms, which implies a high value of q. If $MPK <$ cost of capital, then firms are incurring losses, so their stock market values fall, so $q$is low.

Proposition 7.2.2.1 (Stock Market v.s. GDP)**.** Reasons for a relationship between the stock market and GDP:

1. A wave of pessimism about future profitability of capital would
   (a) cause stock prices to fall
   (b) cause Tobin's q to fall
   (c) shift the investment function down
   (d) cause a negative aggregate demand shock
2. A fall in stock prices would
   (a) reduce household wealth
   (b) shift the consumption function down
   (c) cause a negative aggregate demand shock
3. A fall in stock prices might reflect bad news about technological progress and long-run economic growth. This implies that aggregate supply and full- employment output will be expanding more slowly than people had expected.

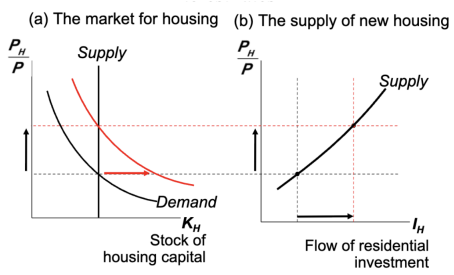
Definition 7.2.3 (Financing Constraints: the fundamental assumption)**.** Neoclassical theory assumes firms can borrow to buy capital whenever doing so is profitable. But some firms face financing constraints: limits on the amounts they can borrow (or otherwise raise in financial markets).A recession reduces current profits. If future profits expected to be high, investment might be worthwhile. But if firm faces financing constraints and current profits are low, firm might be unable to obtain funds. During a credit crunch, financing constraints become *more prevalent* and investment spending *decreases*.

7.3 Residential Investment

Definition 7.3.1 (Residential Investment)**.** Purchases of new housing units (either by occupants or landlords). In this model, we assume

1. The flow of new residential invest, denoted, I_H depends on the relative price of housing $\frac{P_H}{P}$
2. $\frac{P_H}{P}$ is determined by supply and demand in the market for existing houses.

Proposition 7.3.1.1 (Response to a fall in interest rates)**.** Residential investment responds to a fall in interest rates by having a higher demand for houses (since the opportunity cost for housing capital increases).



7.4 Inventory Investment

Definition 7.4.1 (Inventory Investment)**.** The value of the change in inventories of finished goods, materials and supplies, and a work in progress.

Remarks 7.4.1.0.1. Keynes believes that the main driving force of the Business Cycle is inventory investment. However, this is generally not true, as it only accounts for 1% of the GDP.

Proposition 7.4.1.1 (Motives for holding inventories)**.**

1. *Production Smoothing.* Sales fluctuate, but many firms find it cheaper to produce at a steaty rate:
   (a) When sales < production, inventories rise.
   (b) When sales > production, inventories fall.
2. *Inventories as a factor of production* Inventories allow some firms to operate more efficiently: 1) samples for retail sales purposes, 2) spare parts for when machines break down.
3. *Stock-Out Avoidance*: To prevent lost sales when demand is higher than expected.
4. *Work in Process*: Goods not yet completed are counted in inventory.

Definition 7.4.2 (The Accelerator Model)**.** A simple theory that explains the behavior of inventory investment, without endorsing any particular motive.

Model Derivation. Let N denote that stock of inventories, ΔN the inventory investment. Assume firms hold a stock of inventories proportional to their output,

$$N = \beta Y,$$

where β is an exogenous parameter reflecting firms' desired stock of inventory as a proportion of output. Then,

$$\Delta N = \beta \Delta Y = I_{inventory},$$

i.e., inventory investment is proportional to the change in output. Thus,

1. When output is rising, firms increase inventories
2. When output is falling, firms allow their inventories to run down.

□

Definition 7.4.3 (Inventories and the Real Interest Rate)**.** However, many firms have moved toward *just-in-time production* due to the opportunity cost of holding goods in inventory: the interest that could have been earned on the revenue from selling those goods. Hence, inventory investment depends on the real interest rate.
Example. High interest rates in the 1980s motivated many firms to adopt just-in-time production, which is designed to reduce inventories.

Remarks 7.4.3.0.1 (Summery)**.** Keynes posits that

$$I = I_{fixed} + I_{residental} + I_{Inventory} \text{ and } I(r^{-}).$$

Investigate further, it is shown that I_{fixed} and $I_{residential}$ drive the investment mainly and has, as well, inverse relationship with r.

8 Micro-Basis: Money

8.1 Money Supply

Definition 8.1.1 (Money Supply)**.** The money supply equals currency plus demand (checking account) deposits:

$$M = C + D.$$

Since the money supply includes demand deposits, the banking system plays an important role.

Definition 8.1.2 (Reserve)**.** Suppose the *portion of deposits that banks have not lent is* R. A bank's liabilities include deposits, assets include reserves and outstanding loans.

1. The 100% *reserve banking system* is a system in which banks hold all deposits as reserves.

2. The *fractional reserve banking system* is a system in which banks hold a fraction of their deposits as reserves.

Proposition 8.1.2.1 (Theory Development)**.** Suppose there is no banks and

$$D = 0 \text{ and } M = C = 1000.$$

Now in the 100% *reserve banking system*, with households deposit the \$1000 at the bank. The balance sheet is given

Assets	Liabilities
Reserve \$1000	Deposits \$1000

after this deposit

$$C = 0, M = \$1000 = D.$$

Say, we are in the *fractional reserve banking system* that requires \$20 if the deposits in reserve. Then, the balance sheet will be

First Bank	
Assets	Liabilities
Reserve \$200	Deposits \$1000
Loans \$800	

in which

$$M = 1000 + 800 = 1800; C = 800 \text{ and } 1000 = D.$$

Now say the borrowed 800 is deposited into a second bank with the same reservation ratio,

Second Bank	
Assets	Liabilities
Reserve \$160	Deposits \$800
Loans \$640	

Finally, suppose the 640 is eventually deposited in Third Bank,

Third Bank	
Assets	Liabilities
Reserve \$128	Deposits \$640
Loans \$512	

And so the total amount of money is given

+	Original Deposit	$= 1000$
+	First Bank Lending	$= 800$
+	Second Bank Lending	$= 640$
+	Third Bank Lending	$= 512$
+	Other Lending...	

And so we can write a general formula. Let $rr \in (0, 1]$ be the reserve ratio. Let the initial deposit be $\mathscr{D}$. Then the total amount of money $\mathscr{M}$ is given by

$$\mathscr{M} = \mathscr{D}\left(1 + (1 - rr) + (1 - rr)^2 + \dots\right) = \sum_{n=0}^{\infty} \mathscr{D}\,(1 - rr)^n = \frac{\mathscr{D}}{1 - (1 - rr)} = \frac{\mathscr{D}}{rr},$$

by the geometric test. Note that if $rr = 0$, then $\mathscr{M} = \infty$.

Definition 8.1.3 (Money Supply Model)**.** We have exogenous variables

1. *Monetary base,* controlled by central bank,
$$B = C + R.$$
2. *Reserve-deposit ratio*, depends on regulations and bank policies,
$$rr = \frac{R}{D}.$$
3. *Currency Deposit Ratio,* depends on households' preferences,
$$cr = \frac{C}{D}.$$

Proposition 8.1.3.1 (Solving for the money supply)**.** Let

$$M = C + D = \frac{C+D}{B} \times B = m \times B,$$

where

$$m = \frac{C+D}{B} = \frac{C+D}{C+R} = \frac{\frac{C+D}{D}}{\frac{C+R}{D}} = \frac{cr+1}{cr+rr},$$

in which B is policy variable

Definition 8.1.4 (Money Multiplier)**.**

$$M = m \times B \text{ s.t. } m = \frac{cr+1}{cr+rr}.$$

If $rr < 1, m > 1$. If monetary base change by ΔB, then

$$\Delta M = m \times \Delta B,$$

note that m is the money multiplier, the increase in the money supply resulting from a one-dollar increase in the monetary base.

Definition 8.1.5 (Three instruments of Monetary policy)**.**

1. *Open Market Operations:* the purchase or sale of government bonds by Bank of Canada. If the Bank buys bonds from the public, it pays with new dollars, increasing $B \implies M$.

2. *Deposit Switching:* Bank of Canada holds large bank deposits. Can hold the either at the Bank itself or at charted Banks. Deposit switch increases charted banks reserves.

3. *The discount (Bank) rate:* the interest rate that the Central Bank charges on loans it makes to banks. When banks borrow from the Bank, their reserves increase, allowing them to make more loans and "create" more money. The Bank can increase B by lowering the discount rate to induce banks to borrow more reserves.

In short, Open Market Operation is most frequently used, followed by deposit switching. Changes in the discount rate is however largely symbolic as The Bank is *a lender of last resort.*

Proposition 8.1.5.1 (Bank can't precisely control M)**.** It is noteworthy that the Bank cannot control M precisely. Recall that $M = m \times B$ s.t. $m = \frac{C+D}{B} = \frac{cr+1}{cr+rr}$. Households can change cr, causing m and M to change. Thus, Banks often hold *excess reserves,* if banks change their reserves, then rr, m and M change.

8.2 Case Study: Bank Failures

From 1929 to 1933, over 9000 banks closed and money supply fell 28 %. This drop in the money supply may have caused the great depression. It certainly contributed to the severity of the depression. Now it is to our inquiry to ask given

$$M = m \times B \text{ s.t. } m = \frac{cr + 1}{cr + rr},$$

why would the central bank allow so? It can be argued that the reason for the failure is twofold:

1. loss of confidence in bank (change in consumer preference)

$$cr \uparrow \rightarrow m \downarrow$$

2. Banks became more cautious

$$rr \uparrow \rightarrow m \downarrow .$$

In 1929, $m = 3.7, B = 7.1; M = 26.5$. In 1933, $m = 2.3, B = 8.4; M = 19$.

Bank failures in the 1930s

	August 1929	*March 1933*	*% change*
M	26.5	19.0	–28.3%
C	3.9	5.5	41.0
D	22.6	13.5	–40.3
B	7.1	8.4	18.3
C	3.9	5.5	41.0
R	3.2	2.9	–9.4
m	3.7	2.3	–37.8
rr	0.14	0.21	50.0
cr	0.17	0.41	141.2

Many policies have been implemented since the 1930s to prevent such widespread bank failures. E.g., Federal Deposit Insurance, to prevent bank runs and large swings in the currency- deposit ratio.Yet a more or less similar problem happened in 2008-2009.

Definition 8.2.1 (Bank Capital)**.** The resources a bank's owners have put into the bank. This gives a more realistic balance sheet:

Assets	Liabilities and Owners' Equity
Reserves \$200	Deposits Reserves \$750
Loans\$500	Debt \$200
Securities \$300	Capital (owner's equity) \$50

Definition 8.2.2 (Leverage)**.** The use of borrowed money to supplement existing funds for purposes of investment. We define

$$leverage\,ratio = \frac{asset}{capital} = \frac{200 + 500 + 300}{50} = 20.$$

Remarks 8.2.2.0.1. Being highly leveraged make the bank vulnerable. Sps a recession causes our bank's assets to fall by 5% to 950. Then, capital = assets − liabilities = 0. In short 1928 accident was caused by bank run ($cr, rr \uparrow$). Whereas in 2008, $\overline{cr}, rr \uparrow$.

Definition 8.2.3 (Capital Requirement)**.** Minimum amount of capital mandated by regulators; intended to insure that banks will be able to pay off depositors; higher for banks that hold more risky assets. In the 08-09 financial crisis, we have losses on mortgages shrunk bank capital, slowed lending, exacerbated the recession; Govt injected billions of capital into banks to ease crisis and encourage more lending.

8.3 Money Demand

Definition 8.3.1 (Portfolio Theories)**.** Emphasize *store of value function of money:* in particular, M2, M3.

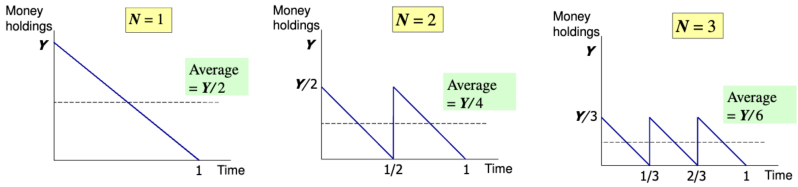
Definition 8.3.2 (Transactions Theories)**.** Emphasize *medium of exchange function:* in particular, M1.

Proposition 8.3.2.1 (A simple profolio theory)**.** We have

$$(M/P)^d = L\left(r_s^-, r_b^-, \pi_-^e, W_+\right)$$

where r_s = *expected real return on stocks* r_b = *expected real return on bonds* π^e = *expected inflation rate*, W = *real wealth.*

Proposition 8.3.2.2 (The Baumol-Tobin Model:transactions theory)**.** Let Y be *total spending, done gradually over the year,* i be the *interest rate on sabvings account,* N be the *number of trips consumer makes to the bank to withdrew from savings account,* and F be *the cost of a trip to the bank (e.g., if a trip takes 15 minutes and consumer's wage is* 12*, then ,*$F = 3$*).* Then, the money holdings over a year is

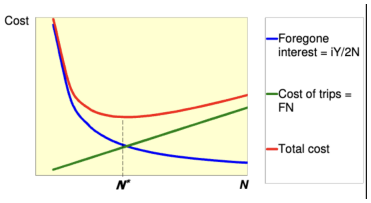


On average, an individual holds average money $= Y/2N$ forgoing interest rate $i \times (Y/2N)$ and spends $N \times F$ for travling to the bank. Thus,

$$TC = i \times \frac{Y}{2N} + F \times N.$$

Given Y, i, F an individual optimize through her choice of N :

$$\frac{\partial TC}{\partial N} \overset{optimize}{=} 0 = -\frac{iY}{2N^2} + F \implies N = \sqrt{\frac{iY}{2F}}.$$



And so subsitution for average money holding yields,

$$Y/2N = \sqrt{\frac{YF}{2i}}.$$

Thus, we have

Definition 8.3.3 (The Baumol-Tobin money demand function)**.** The Baumol-Tobin money demand function is given by

$$(M/P)^d = \sqrt{\frac{YF}{2i}} = L(i, Y, F).$$

B-T shows how F affects money demand. B-T implies that income elasticity of money demand $= 0.5$, interest rate elasticity of money demand $= -0.5$.

Remarks 8.3.3.0.1 (Examples of financial innovation)**.**

1. Many checking accounts now pay interest
2. Mutual funds are baskets of stocks that are easy to redeem- just write a check
3. non-monetary assets having some of the lquidity of money are called *near money*
4. money and near money are close substitutes, and switching from one to the other is easy.

The rise of near money makes money demand less stable and complicates monetary policy. 1993: the Fed switched from targeting monetary aggregates to targeting the Federal Funds rate.This change may help explain why the U.S. economy was so stable during the rest of the 1990s. In other words,

$$MV = PQ$$

but V is unknown, and so we target PQ, in terms of r.

9 Advances in Business Cycle Theory

9.1 Real Business Cycle Theory

Remarks 9.1.0.0.1. In which framework, prices are fully flexible, even in the short-run. Stabilization policy must show *real* effects. It builds on

1. Interpretation of the labor market.
2. Importance of technology shock.
3. Neutrality of Money.
4. Wage and price flexibility.

Proposition 9.1.0.1 (Reasoning of the model)**.** Suppose we are in an isolated island. Then, we divides our time into leisure and working (production and investment) from which we optimize. Shocks to us are natural. For example, earthquake, etc. Our labor of working is thus predicated on such shock. Say we produce agricultural goods. Since the earthquake lower our productivity, it is not as desirable for us to have worked. Thus, $L, y, I \downarrow$. We trade working with leisure as working is not as productive.

Remarks 9.1.0.1.1 (The debate over RBC theory)**.** Four issues arise.

1. Do changes in employment reflect voluntary changes in labor supply?
2. Does the economy experience large, exogenous productivity shocks in the short run?
3. Is money really neutral in the short run?
4. Are wages and prices flexible in the short run? Do they adjust quickly to keep supply and demand in balance in all markets?

Definition 9.1.1 (Intertemporal Substitution of Labour)**.** In RBC theory, workers are willing to reallocate labor over time in response to changes in the reward to working now versus later, i.e.,

$$\frac{(1+r)\,W_1}{W_2},$$

where W_1 is the wage in period 1, and W_2 is the wage in period 2.

Proposition 9.1.1.1. Thus, in RBC theory,

1. shocks cause fluctuations in the intertemporal relative wage
2. workers respond by adjusting labor supply
3. this causes employment and output to fluctuate

Although, critics argue that

1. labor supply is not very sensitive to the intertemporal real wage
2. high unemployment observed in recessions is mainly involuntary

Theorem 9.1.1.2 (Technology Drives Growth)**.** In RBC theory, economic fluctuations are caused by productivity shocks. *Solow residual* is defined to be a measure of productivity shocks, shows the change in output that cannot be explained by changes in capital and labor. Given a cobb doglas function

$$Y = AK^{\alpha}L^{\beta}$$

we can take derivative with respect to t as follows:

$$\begin{aligned}
\ln(Y) &= \ln\left(AK^{\alpha}L^{\beta}\right) \\
&= \ln A + \alpha \ln K + \beta \ln L \\
\implies \frac{\Delta Y}{Y} &= \frac{\Delta A}{A} + \alpha\frac{\Delta K}{K} + \beta\frac{\Delta L}{L} && \text{differentiate wr to } t \\
\implies \frac{\Delta A}{A} &= \frac{\Delta Y}{Y} - \alpha\frac{\Delta K}{K} - \beta\frac{\Delta L}{L}
\end{aligned}$$

From which it follows that *the growth of technology causes the growth of output.*

Remarks 9.1.1.2.1. Proponents of RBC theory argue that the strong correlation between output growth and Solow residuals is evidence that productivity shocks are an important source of economic fluctuations. Critics note that the measured Solow residual is biased to appear more cyclical than the true, underlying technology.

Proposition 9.1.1.3 (The neutrality of money)**.** RBC critics note that reductions in money growth and inflation are almost always associated with periods of high unemployment and low output. RBC proponents respond by claiming that the money supply is endogenous: Suppose output is expected to fall. Central bank reduces money supply in response to an expected fall in money demand.

Proposition 9.1.1.4 (Wage and price flexibility)**.** RBC theory assumes that wages and prices are completely flexible, so markets always clear. RBC proponents argue that the degree of price stickiness occurring in the real world is not important for understanding economic fluctuations. RBC proponents also assume flexible prices to be consistent with microeconomic theory. Critics believe that wage and price stickiness explains involuntary unemployment and the non-neutrality of money.

9.2 New Keynesian Economics

Remarks 9.2.0.0.1. In which framework wages and prices are sticky in the short-run. The theory advocates for Aggregate Demand Management (IS-LM) as a key to economic stability. Most economists believe that short-run fluctuations in output and employment represent deviations from the natural rate, and that these deviations occur because wages and prices are sticky. New Keynesian research attempts to explain the stickiness of wages and prices by examining the microeconomics of price adjustment.

Proposition 9.2.0.1 (Recession as Coordination Failure)**.** Each firm must decide whether to cut prices after a decline in the money supply. Firms make this decision without knowing the strategy other firms choose. Inferior outcomes due to coordination failure would cause a recession. Consider the following example. Two firms: 1 and 2. Two strategies: Cut Price, Keep High price. Firm 1's best strategy is Cut Price to make highest possible profit. Firm 2's best strategy is Cut Price to make highest possible profit.

Where the Nash is (Cut, Cut) , (Keep, Keep) . However, coordinated price reduction is prohibited by Anti-Trust Law.

Proposition 9.2.0.2 (Staggering Price Variations)**.** Staggering makes the overall level of prices adjust gradually, even when individual prices change frequently. Firms change prices intermittently in response to a demand shift and change in profit. Prices change in the beginning, meddle, and end of the month.

Proposition 9.2.0.3 (Staggering Wage Variations)**.** A decline in the money supply reduces the level of Aggregate Demand, output, and employment. Lower employment requires nominal wage rate to fall. But, workers and labor unions are reluctant to take the wage cut. The reluctance of a worker to be the first to take a pay cut makes the overall level of wages slow to respond to changes in the Aggregate Demand

Proposition 9.2.0.4 (Top Reasons for Sticky Prices)**.** A survey of managers show that the top reasons for sticky prices are

1. Coordination failure: firms hold back on price changes, waiting for others to go first
2. Firms delay raising prices until costs rise
3. Firms prefer to vary other product attributes, such as quality, service, or delivery lags
4. Implicit contracts: firms tacitly agree to stabilize prices, perhaps out of 'fairness' to customers
5. Explicit contracts that fix nominal prices
6. Menu costs

Remarks 9.2.0.4.1 (Frontier of Research)**.** Two distinct approaches to the study of business cycles: Real Business Cycle theory v.s. New Keynesian theory. Not all economists fall entirely into one camp or the other. An increasing amount of research incorporates insights from both schools of thought to advance our understanding of economic fluctuations.