

Q5.

$$h(x) = \frac{x+3}{x+2}$$

Solution.

First, we note that $h(x)$ can be 'decomposed' into two functions. The first one being $x + 3$, the second, $x + 2$.

By the quotient rule,

$$h'(x) = \frac{(x+3)'(x+2) - (x+3)(x+2)'}{(x+2)^2} \text{ where ' refers to the 'derivative of'}$$

Thus, as $(x + 3)' = 1$ and that $(x + 2)' = 1$

ork Solution:

$$\text{We have that } h'(x) = \frac{1 \cdot (x+2) - (x+3) \cdot 1}{(x+2)^2} = \frac{(x+2) - (x+3)}{(x+2)^2} \text{!!!!!!!!!!}$$

Q6. (advised)

$$g(x) = (x^2 + 7)^4$$

$$\begin{aligned} 20^0 &= 20^1 / 20 = 1 \\ 20^1 &= 20^2 / 20 \\ 20^2 &= 20^3 / 20 \\ 20^3 &= 20^4 / 20 \end{aligned}$$

Power Rule:

$f(x) := ax^n + c$ st. $a, c, n \in \mathbb{R}$,
then

$$f'(x) = anx^{n-1} + 0 = anx^{n-1}$$

Note that if

$g(x) = c$ where c is any number
then

$$g'(x) = 0.$$

say $h(x) = 2$

because 2 is a number,

$$h'(x) = 2 = 0.$$

The power rule allows us to differentiate a polynomial easily.

In particular,

$$(x + 3)' = x' + 3' \text{ and } \frac{d(x)}{dx} = 1 \cdot x^{1-1} = 1 \cdot x^0 = 1 \cdot 1 = 1$$

Secondly!! $\frac{d(3)}{dx} = 0$ because 3 is not dependent on x (and we are talking about 'with respect to x ').

So together by 1 and 2 we have

$$(x + 3)' = 1 + 0 = 1 \text{ Note that } (x + 3)' \equiv \frac{d(x+3)}{dx} \equiv \frac{df}{dx} \text{ when } f = (x + 3)$$

Exercise:

What is $(x + 2)'$

We can think this in two parts. x and 2

Consider x .

$$\frac{d}{dx}(x) = 1; \text{ the second part } \frac{d}{dx}(2) = 0$$

$$\frac{d}{dx}(x + 2) = (x + 2)' = \frac{d}{dx}(x) + \frac{d}{dx}(2) = 1 + 0 = 1$$

$$\frac{d}{dx}(c)$$

|||

$$\frac{dg}{dx}$$

+ 3, the second, $x + 2$.

Side notes. Axiom: let $c \in \mathbb{R}$, then $c^0 = 1$.

The reason is this. Consider 20.

$$20^0 = 1$$

$$20^1 = 20$$

$$20^2 = 20 * 20$$

$$20^3 = 20 * 20 * 20$$

$$\vdots$$

$$20^n = 20 * \dots * 20 \text{ for } n \text{ times}$$

Note that for any integer n ,

$$20^{n-1} = 20^n / 20$$

The intuition is that consider $n = 3$

Then it's obvious!! Because

$$20^2 = 20 * 20 = 20 * 20 * 20 / 20 = 20^3 / 20$$

Hence, for 20^0 it must be that

$$20^0 = 20^1 / 20 = 1$$

Today!!! We are going to talk about 'MARGIN' in (micro)economic!!!

Minds ON:

What does margin means? In economics we often hear phrases like

'marginally diminishes (decreasing)' What are economist talking about??

For economists it happens to be the case that margin is exactly derivative!!

First, we start with the demand curve.

Supply is from PRODUCERS

Demand Curve is the UTILITY FUNCTION. UTILITY can be thought of as the happiness. In this case happiness of the consumer.

Demand is from CONSUMERS

UTILITY FUNCTION is defined as follows: $U(x) = x^\alpha$ where $\alpha - 1 < 0$. Where x is a good. For example, x can be say apple.

When we take derivative of U we get something called the marginal utility function!!! that is,
 $U'(x) = MU(x) = \alpha x^{\alpha-1}$.

We usually say that $MU(x)$ is 'marginally diminishing' because
 $MU'(x) = \alpha \cdot (\alpha - 1)x^{\alpha-2} < 0$ In calculus this means 'decreasing for all input'.

Lets work with an example.

Consider $U(x) = -x^2 + 500x$ what will be MU.

$$MU = d/dx(-x^2 + 500x) = 500 - 2x$$

First we check if the law of marginal diminishing utility holds:

Note that

$$d/dx(MU) = -2 < 0$$

so U is indeed a utility function.

Think x as q then $MU = 500 - 2q$. In particular in economic theory we have

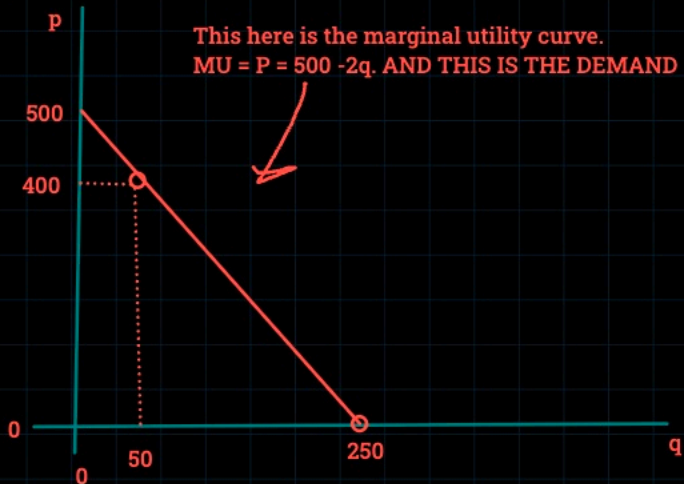
$$MU = \text{price} = P$$

So we have

$$P = 500 - 2q. \text{ (graphically on the left)}$$

$$0 = 500 - 2q \implies 2q = 500 \implies q = 250.$$

Note: P is the average price.



One is tempted to ask how the hell is $U(x)$ defined to be like this???

ANSWER:

- 1) it was originally pure theory.
- 2) we collect data and showed causality in econometric (statistics)

$$\hat{U} = \beta_0 + \beta_1 x + \beta_2 x^2 \text{ given a good } x$$

In particular, it is the case that

$$\beta_1 > 0 \text{ and } \beta_2 < 0$$

We usually however samples the marginal utility because data on price and quantity is easier to be found.

We have

$$\hat{Q} = a - bP \implies P = a/b - \hat{Q}/b.$$

Summary:

- 1) Margin is Derivative!!**
- 2) Utility (happiness) is marginally diminishing**
- 3) How did we get utility function? From statistics (regression model)**