

(b) $y = \sqrt{2x + 1}$

Solution. First note that y is again a combination of two functions. In particular it is a composite function. Let $f(x) = \sqrt{x} = x^{1/2}$; $g(x) = 2x + 1$. Then, $y = f(g(x))$. By power rule,

$$f'(x) = \frac{1}{2}x^{-1/2} \text{ and } g'(x) = 2.$$

Now we check our formula for chain rule. It states that $y' = f'(g(x)) \cdot g'(x)$. Thus, we have

$$y' = \frac{1}{2}(2x + 1)^{-1/2} \cdot 2 = \frac{1}{\sqrt{2x + 1}}.$$



$$(b) \quad y = \frac{\sin x}{x}$$

Solution. Observe that y is a quotient. Let $f(x) = \sin x$; $g(x) = x$.
 $f'(x) = \cos(x)$ by the note. $g'(x) = 1$ And so by quotient rule, we have

$$y' = \frac{\cos(x)x - \sin(x)}{x^2}.$$



(b) $y = x^2 e^x$

Solution. First note that y is a product of two functions.

Let $f(x) = x^2$ and $g(x) = e^x$. Furthermore, note that by power rule and hint we have:
 $f'(x) = 2x$ and $g'(x) = e^x$. By product rule,

$$y' = (x^2)'e^x + x^2(e^x)' = 2xe^x + x^2e^x.$$



(a) $f(x) = x^5 - 3x^2 + 7$

Solution. Let's apply power rule: we get

$$f'(x) = 5x^4 - 6x.$$

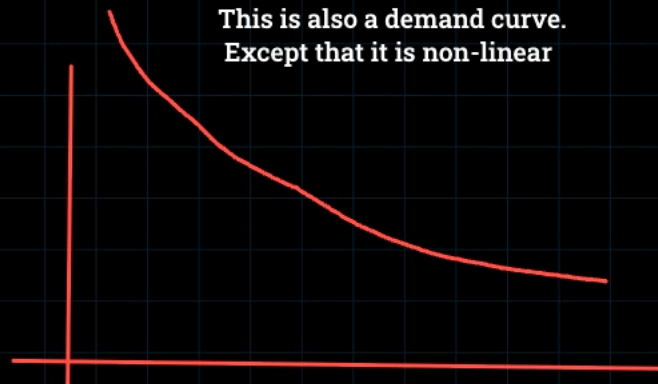
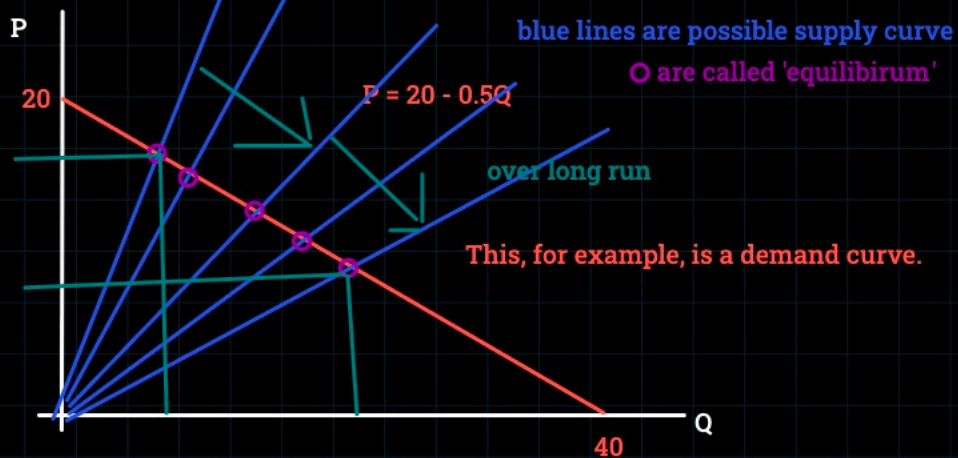


To Do:

Supply Side Theory.

Review: Demand?

1. Demand is marginally diminishing
2. Demand is the marginal utility
3. Utility just is happiness and everyone's utility curve is different



We also relate price with utility. Consumer pays the last unit happiness: marginal utility. Therefore, to them price, $P = MU$ (marginal utility) = a function of quantity.

Now consider the short run.

WE now want to explain why $P = MC$.

THE IDEA IS THAT

producers are RATIONAL. BY this, it means they maximize profit: i.e., they optimize.

WHAT is PROFIT?

Recall profit = Price - Cost

Total profit: <---- very bad term

we usually call this thing TOTAL REVENUE

TOTAL REVENUE: = Price \cdot Quantity

For example: price = 10 and quantity = 3 then total revenue = 30

On the other hand we have this thing called total cost.

Total cost is a function of quantity.

TOTAL PROFIT = TOTAL REVENUE - TOTAL COST

Let profit be Π ; TOTAL REVENUE = TR; TOTAL COST = TC

SO
 $\Pi = TR - TC = PQ - TC$ for the producer.

In Calculus, we maximize or minimize where relevant.

In particular, taking derivative and setting so to be 0 achieve this goal

In a very non-rigours way.

TC

We want to find Q such that Π is maximizes:

in the other words, $\Pi' = \frac{d\Pi}{dQ} = 0$.

Note that $(PQ)'$ with respect to Q is P
(because we are price taker).

Also $(TC)' = \text{Marginal Cost} = MC$

SO!!!

$\frac{d\Pi}{dQ} = 0 = P - MC$ by power rule of calculus

SO it follows that


$P = MC$ holds, in short run.

Question: why 0?


1 “ Fermat's Theorem X

2 “ Take this fact for now as a coincidence. In fact X

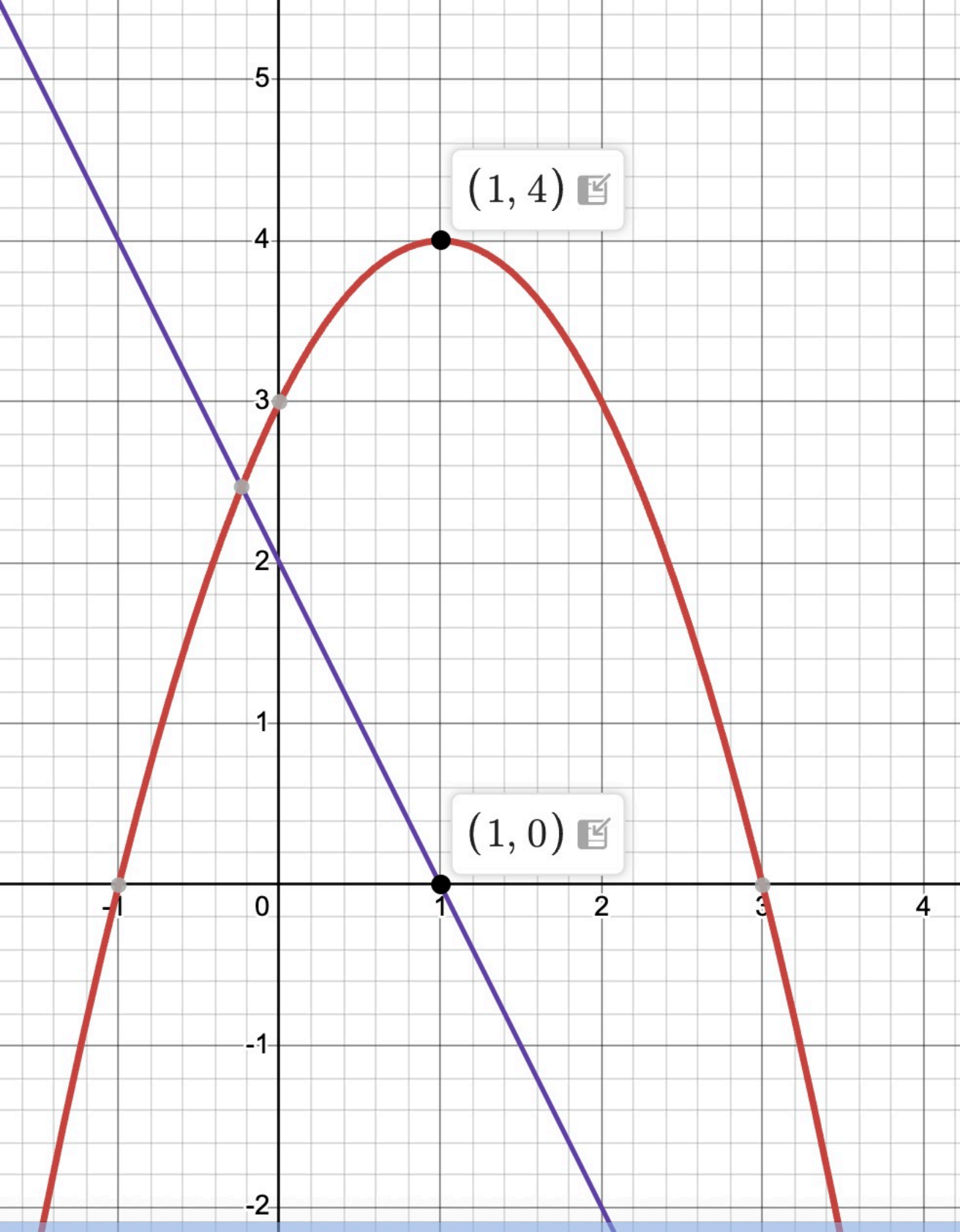
3 “ In fact this is something called extrema theorem X

4  $y = -x^2 + 2x + 3$ X

5 “ the derivative X

6  $y = -2x + 2$ X

7 “ This is a reason why we set derivative = 0 to 'optimize', i.e., minimize/maximize | X



Today we will be talking about something called the supply curve.
In particular, we relate supply to the rationality of producer.
We assume that the producer aims to maximizes profit.

First let's define the total cost of a producer.

Let the total cost function be TC : = a function of quantity Q .

Note that if a producer produces more, its total cost tends to increase. There is also a constant fee we shall categorize as 'fixed cost.'

For example.

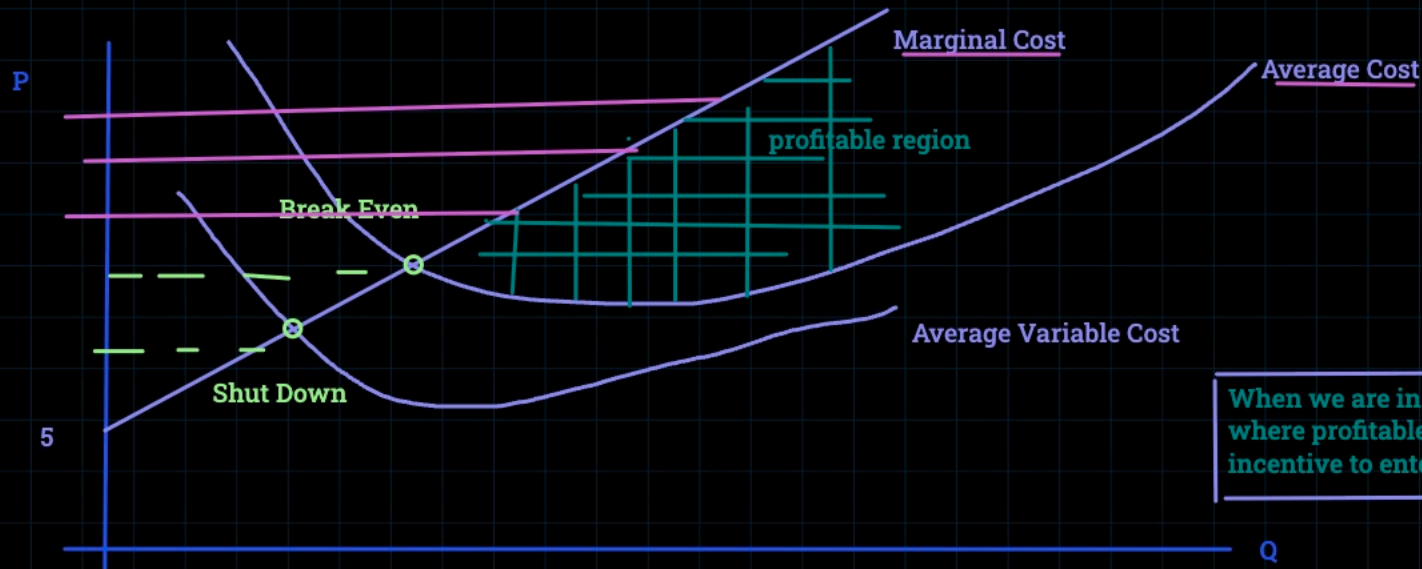
$$TC(q) = 10q^2 + 5q + 200.$$

Then, the 200 is called the fixed cost. Where as $10q^2 + 5q$ is called the 'variable cost'

Here abstract price as if cost: also since price is per unit, we consider the average cost, average variable cost in our case

Since it is a competitive market, it has no pricing power. The producer takes the price. His profit can be found on a chart.

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TOTAL PROFIT = TOTAL REVENUE - TOTAL COST
Let profit be Π ; TOTAL PROFIT = TOTAL REVENUE - TOTAL COST
SO
 $\Pi = TR - TC = P \cdot Q - TC$

When we are in a competitive market, where profitable region exists; producers will have incentive to enter the market.

In short run however we have price = MC

Furthermore, we define marginal cost: in this case $TC'(q) = MC(q) = 20q + 5$

Average cost = $\frac{TC}{Q} = 10q + 5 + \frac{200}{q}$

Average variable cost = $\frac{VC}{Q} = 10q + 5$

~~IN LR!!! MC == AC no one earns profit.~~

○ is called

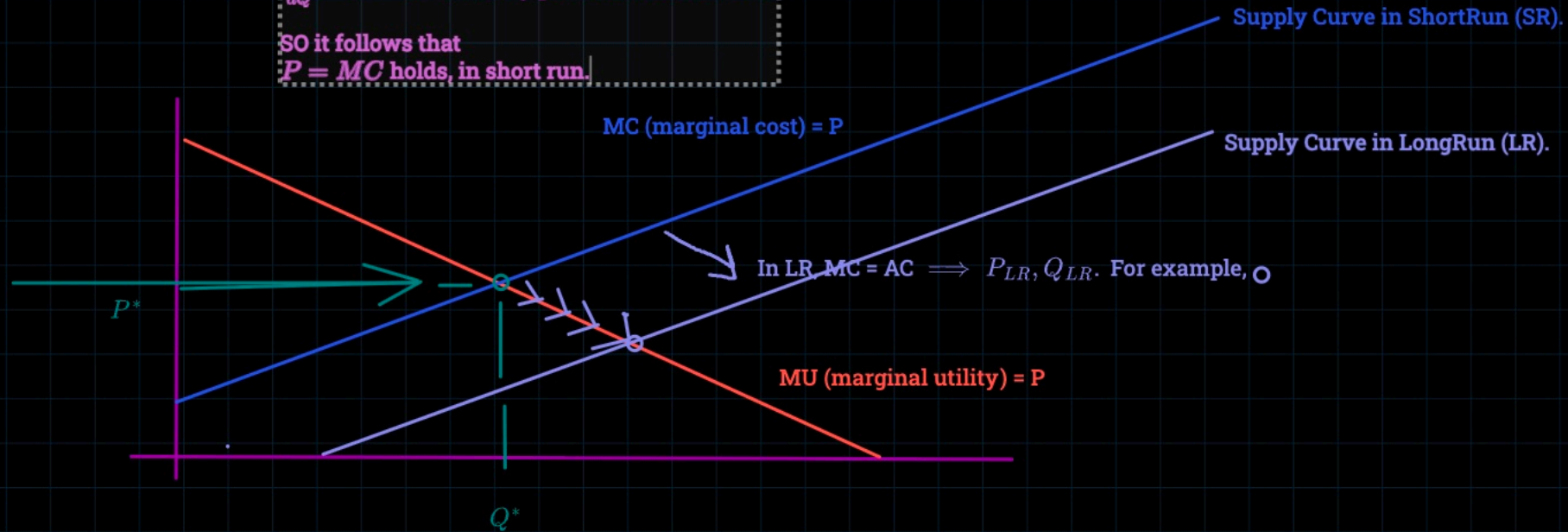
or we have

$$\frac{d\Pi}{dQ} = 0 = P - MC \text{ by power rule of calculus}$$

So it follows that

$P = MC$ holds, in short run.

○ is called the equilibrium



At P^*, Q^* we have a equilibrium. This is a point where the market is stable.